

Connect the Dots

Use vertices, edges, degrees, and paths to analyze networks.

HIGH SCHOOL • MA.912.LT.2.1

MA.912.LT.2.1 "Define and explain the basic concepts of Graph Theory."

NAME _____

DATE _____

GRAPH CONNECTIONS

A graph uses vertices to represent objects and edges to represent connections. The degree of a vertex is the number of edges that touch it, and a path follows connected edges without repeating a vertex. In a complete graph, every pair of vertices is connected, and the sum of all degrees is twice the number of edges.

WORKED EXAMPLE

A graph has 8 vertices, and each vertex has degree 6. How many edges does the graph have?

1. Add the degrees: $8 \times 6 = 48$.
2. Each edge is counted at both endpoints.
3. Divide by 2: $48 / 2 = 24$.

Answer: 24 edges

PRACTICE 6 problems

- 1** Graph G has vertices A, B, C, D, E, F, and G. Its edges are AB, AC, AD, BE, BF, CD, CE, DG, and EF. How many edges does Graph G have?

ANSWER _____

- 2** In a graph, vertex R is joined directly to A, B, C, D, and E. What is the degree of R?

ANSWER _____

- 3** A graph has edges PQ, QR, RS, ST, and TP. Is P-Q-R-S a path?

ANSWER _____

- 4** The sum of the degrees of all vertices in a graph is 36. How many edges does the graph have?

ANSWER _____

- 5** A complete graph has 7 vertices. How many edges does it have?

ANSWER _____

- 6** A graph has edges UV, VW, XY, and YZ. Is the graph connected?

ANSWER _____

APPLY IT Show your work

- 1** A school music club maps who can share rehearsal files directly. The connections are Ari-Bea, Ari-Cam, Ari-Dee, Bea-Cam, Cam-Eli, and Dee-Eli. Which members have the greatest number of direct sharing partners, and what is that degree?

WORK SPACE

ANSWER _____

- 2** Students are organizing a gaming meetup. A direct message link exists between Nia and Luis, Nia and Priya, Luis and Omar, Luis and Tessa, Priya and Tessa, Omar and Zoe, and Tessa and Zoe. What is the fewest number of direct-message links Nia must use to reach Zoe? Give one shortest path.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A network graph is said to have 9 vertices, with every vertex having degree 5. Can this graph exist? Explain your reasoning.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Graph G has vertices A, B, C, D, E, F, and G. Its edges are AB, AC, AD, BE, BF, CD, CE, DG, and EF. How many edges does Graph G have?	9 edges
2	In a graph, vertex R is joined directly to A, B, C, D, and E. What is the degree of R?	5
3	A graph has edges PQ, QR, RS, ST, and TP. Is P-Q-R-S a path?	Yes, P-Q-R-S is a path.
4	The sum of the degrees of all vertices in a graph is 36. How many edges does the graph have?	18 edges
5	A complete graph has 7 vertices. How many edges does it have?	21 edges
6	A graph has edges UV, VW, XY, and YZ. Is the graph connected?	No. There is no path from V to Y.

PART 2 • APPLY IT

1. **Ari and Cam, each with degree 3.** Count the edges touching each member. Ari connects to Bea, Cam, and Dee, while Cam connects to Ari, Bea, and Eli.
2. **3 links, such as Nia-Luis-Omar-Zoe.** Find a path from Nia to Zoe with the fewest edges. Nia-Luis-Omar-Zoe uses 3 links, and no route uses fewer.

CHALLENGE

No. The sum of the degrees would be $9 \times 5 = 45$, which is odd. A graph's degree sum must be even because every edge contributes 2 to the sum.

Accept any correctly named shortest path in the gaming meetup problem. For the challenge, accept any explanation that states the degree sum must be even.