

Color Code Quest

Use vertex colors to organize conflicts.

HIGH SCHOOL • MA.912.LT.2.4

MA.912.LT.2.4 "Apply graph coloring techniques to solve problems."

NAME _____

DATE _____

COLORING A GRAPH

A proper vertex coloring gives different colors to vertices connected by an edge. The chromatic number is the least number of colors that makes a proper coloring possible. Look for groups in which every vertex connects to every other vertex, because those groups must all have different colors.

WORKED EXAMPLE

Vertices A, B, C, D, E, and F have edges AB, BC, CA, CD, DE, and EF. Find the least number of colors needed.

1. A, B, and C form a triangle, so give them red, blue, and green.
2. D touches C, so color D red.
3. E touches D, so color E blue. F touches E, so color F red.
4. Three colors work, and the triangle means two colors cannot work.

Answer: The least number is 3 colors.

PRACTICE 6 problems

- 1** Vertices J, K, L, and M have edges JK, KL, LM, and MJ. What is the chromatic number?

ANSWER _____

- 2** Vertices Q, R, and S have edges QR, RS, and QS. What is the chromatic number?

ANSWER _____

- 3** Vertices V, W, X, Y, and Z have edges VW, WX, XV, XY, and YZ. What is the chromatic number?

ANSWER _____

- 4** Vertices G, H, I, J, K, L, and M form a path with edges GH, HI, IJ, JK, KL, and LM. What is the least number of colors? Give one proper coloring.

ANSWER _____

- 5** Vertices H, J, K, L, M, N, O, and P have edges JK, KL, LM, MN, NO, OP, PJ, plus HJ, HK, HL, HM, HN, HO, and HP. What is the chromatic number?

ANSWER _____

- 6** Vertices X, Y, and Z are each connected to P, Q, R, and S. There are no other edges. What is the chromatic number?

ANSWER _____

APPLY IT Show your work

- 1** For a student music showcase, Imani, Leo, Maya, Noah, and Priya cannot perform in the same time slot when these pairs have a conflict: Imani and Leo, Imani and Maya, Leo and Maya, Maya and Noah, Noah and Priya. What is the least number of time slots? Give one schedule.

WORK SPACE

ANSWER _____

- 2** A club showcase has five events: Film, Robotics, Dance, Gaming, and Debate. These pairs cannot occur at the same time: Film and Robotics, Film and Dance, Robotics and Dance, Robotics and Gaming, Dance and Debate, Gaming and Debate. What is the least number of time slots? Give one schedule.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A graph has vertices Q, R, S, T, and U with edges QR, RS, ST, TU, UQ, and QS. Find the chromatic number and explain why fewer colors cannot work.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Vertices J, K, L, and M have edges JK, KL, LM, and MJ. What is the chromatic number?	2
2	Vertices Q, R, and S have edges QR, RS, and QS. What is the chromatic number?	3
3	Vertices V, W, X, Y, and Z have edges VW, WX, XV, XY, and YZ. What is the chromatic number?	3
4	Vertices G, H, I, J, K, L, and M form a path with edges GH, HI, IJ, JK, KL, and LM. What is the least number of colors? Give one proper coloring.	2 colors; G, I, K, and M can be red, while H, J, and L can be blue.
5	Vertices H, J, K, L, M, N, O, and P have edges JK, KL, LM, MN, NO, OP, PJ, plus HJ, HK, HL, HM, HN, HO, and HP. What is the chromatic number?	4
6	Vertices X, Y, and Z are each connected to P, Q, R, and S. There are no other edges. What is the chromatic number?	2

PART 2 • APPLY IT

- 3 time slots; Imani in slot 1, Leo in slot 2, Maya in slot 3, Noah in slot 1, and Priya in slot 2.** The conflicts among Imani, Leo, and Maya form a triangle, so at least 3 slots are needed. The listed schedule gives different slots to every conflicting pair.
- 3 time slots; Film in slot 1, Robotics in slot 2, Dance in slot 3, Gaming in slot 1, and Debate in slot 2.** Film, Robotics, and Dance form a triangle, so 3 slots are necessary. Assigning Gaming to slot 1 and Debate to slot 2 avoids each remaining conflict.

CHALLENGE

The chromatic number is 3. Q, R, and S form a triangle, so they need three different colors. Color Q red, R blue, S green, T red, and U blue; this is a proper 3-coloring.

Accept any color names or color-number labels that give connected vertices different colors. For items asking for a coloring or schedule, accept any valid assignment using the stated minimum number of colors or slots.