LESSON MAKER PRO

Color the Connections

Use the fewest colors to separate linked vertices.

HIGH SCHOOL · MA.912.LT.2.4

MA.912.LT.2.4 "Apply graph coloring techniques to solve problems."

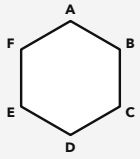
NAME _____ DATE _____

COLORING RULE

In a proper graph coloring, vertices joined by an edge must have different colors. Use as few colors as possible, and reuse a color only on vertices that are not connected.

WORKED EXAMPLE

Use the hexagon as a graph. Each side is an edge. What is the least number of colors needed?

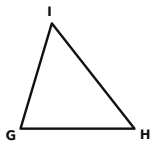


1. Color A red and B blue.
2. Continue alternating red and blue around the ring.
3. F is blue, so it differs from both E and A.

Answer: Two colors: A, C, E red; B, D, F blue.

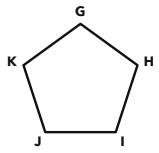
PRACTICE 4 problems

1 Use the triangle as a graph. Each side is an edge. What is the minimum number of colors needed?



ANSWER _____

2 Use the pentagon as a graph. Each side is an edge. Find the minimum colors and give one valid coloring.



ANSWER _____

3 A graph has vertices A, B, C, D. Its edges are AB, AC, AD, BC, BD, and CD. Find the minimum colors and give one coloring.

ANSWER _____

4 A graph has vertices L, M, N, O, P. Its edges are LM, LN, MN, MO, NO, and OP. Find the minimum colors and give one coloring.

ANSWER _____

APPLY IT Show your work

1 At an esports night, each event needs a time slot. Events listed as conflicts cannot share a slot. Find the least number of slots and give a schedule.

ESPORTS EVENT CONFLICTS

Event	Cannot share a slot with
R, Rocket League	S, M
S, Smash	R, M, C
M, Mario Kart	R, S, C
C, Chess	S, M, D
D, Dance game	C

ANSWER _____

2 Five student podcast teams use audio channels. Teams A and B, A and C, B and C, C and D, D and E, and E and A have sound bleed, so each pair needs different channels. What is the least number of channels? Give an assignment.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A graph has vertices A, B, C, D, E. Every pair among A, B, C, D is connected. Vertex E is connected only to A and B. What is the chromatic number? Explain why fewer colors cannot work, then give a coloring.

WORK SPACE AND ANSWER

ANSWER KEY

Color the Connections

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the triangle as a graph. Each side is an edge. What is the minimum number of colors needed?	**3**
2	Use the pentagon as a graph. Each side is an edge. Find the minimum colors and give one valid coloring.	**3 colors; for example, G and I red, H and J blue, K green.**
3	A graph has vertices A, B, C, D. Its edges are AB, AC, AD, BC, BD, and CD. Find the minimum colors and give one coloring.	**4 colors; A red, B blue, C green, D yellow.**
4	A graph has vertices L, M, N, O, P. Its edges are LM, LN, MN, MO, NO, and OP. Find the minimum colors and give one coloring.	**3 colors; L and O red, M and P blue, N green.**

PART 2 · APPLY IT

1. **3 slots; for example, Slot 1: R and C. Slot 2: S and D. Slot 3: M.** R, S, and M form a triangle, so three slots are required. Place C with R and D with S.
2. **3 channels; Channel 1: A and D. Channel 2: B and E. Channel 3: C.** A, B, and C are pairwise connected, so they require three channels. D can share with A, and E can share with B.

CHALLENGE

4 colors. A, B, C, and D are all pairwise adjacent, so they need four different colors. For example, color A red, B blue, C green, D yellow, and E green.

Accept any equivalent color names or color orderings that keep adjacent vertices different. For the challenge, students must state that the four pairwise connected vertices force four colors.