LESSON MAKER PRO

Tree Route Lab

Build efficient connections and follow branching choices.

HIGH SCHOOL · MA.912.LT.2.5

MA.912.LT.2.5 "Apply spanning trees, rooted trees, binary trees and decision trees to solve problems."

NAME _____ DATE _____

TREE TOOLS

A spanning tree includes every vertex, stays connected, and has no cycles, so it has one fewer edge than vertices. In a rooted tree, follow links from the root to find ancestors or distances. In a binary decision tree, each two-choice decision doubles the number of possible endings.

WORKED EXAMPLE

A network has 9 locations. How many links are in any spanning tree for the network?

1. A spanning tree with n locations has n - 1 links.
2. Substitute n = 9.
3. 9 - 1 = 8.

Answer: 8 links

PRACTICE 6 problems

1 A connected graph has 12 vertices. How many edges does one of its spanning trees have?

ANSWER _____

2 In a rooted tree, J is the root. J has children K and L, and L has children M and N. What is the parent of N?

ANSWER _____

3 A full binary tree has a root and four levels below the root. How many leaves are on the lowest level?

ANSWER _____

4 A binary decision tree asks five yes/no questions in order. How many possible ending outcomes are there?

ANSWER _____

5 A weighted graph has vertices A, B, C, and D. Its edges are A-B = 2, A-C = 4, B-C = 1, B-D = 5, and C-D = 3. What is the total weight of a minimum spanning tree?

ANSWER _____

6 A rooted tree has root M. M has children N and O, N has children P and Q, and Q has child R. How many edges are in the path from M to R?

ANSWER _____

APPLY IT Show your work

1 The esports club is placing five computer stations, A, B, C, D, and E. Possible cable routes have these lengths in meters: A-B = 4, A-C = 2, B-C = 1, B-D = 5, C-D = 3, C-E = 6, and D-E = 7. Choose cable routes that connect every station using the least total cable and no loop. What routes should the club use, and what is the total length?

WORK SPACE

ANSWER _____

2 You are planning a student game tournament. You make four decisions, and each decision has two possible choices. Model the choices with a binary decision tree. How many different tournament plans can end at the leaves?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A full binary tree has 31 total vertices. Every non-leaf vertex has exactly two children. How many leaves does the tree have? Explain your reasoning.

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ANSWER KEY

Tree Route Lab

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A connected graph has 12 vertices. How many edges does one of its spanning trees have?	**11 edges**
2	In a rooted tree, J is the root. J has children K and L, and L has children M and N. What is the parent of N?	**L**
3	A full binary tree has a root and four levels below the root. How many leaves are on the lowest level?	**16 leaves**
4	A binary decision tree asks five yes/no questions in order. How many possible ending outcomes are there?	**32 outcomes**
5	A weighted graph has vertices A, B, C, and D. Its edges are A-B = 2, A-C = 4, B-C = 1, B-D = 5, and C-D = 3. What is the total weight of a minimum spanning tree?	**6**
6	A rooted tree has root M. M has children N and O, N has children P and Q, and Q has child R. How many edges are in the path from M to R?	**3 edges**

PART 2 · APPLY IT

1. **Use B-C, A-C, C-D, and C-E for a total of 12 meters.** Choose the shortest routes that do not create a cycle: B-C, A-C, C-D, and C-E. These routes connect all five stations, and 1 + 2 + 3 + 6 = 12.
2. **16 different tournament plans** Each of the four decisions doubles the number of possible paths. The number of leaves is 2 x 2 x 2 x 2 = 16.

CHALLENGE

16 leaves. A tree with 31 vertices has 30 edges. Each non-leaf has two child edges, so there are 15 non-leaf vertices. Then 31 - 15 = 16 leaves.

Accept equivalent route order for the minimum spanning tree in the esports problem. For explanations, accept any correct reasoning that uses the tree edge count and the two-child structure of a full binary tree.