

Strategy Showdown

Find the best choices in zero-sum payoff games.

HIGH SCHOOL • MA.912.LT.2.7

MA.912.LT.2.7 "Solve problems involving optimal strategies in Game Theory."

NAME _____

DATE _____

EQUALIZE THE CHOICES

In a zero-sum payoff matrix, each entry is the payoff to the row player, who wants a larger value. For a mixed strategy in a 2 by 2 game, choose probabilities that make the other player indifferent between choices. Read [a, b; c, d] as Top-Left a, Top-Right b, Bottom-Left c, Bottom-Right d.

WORKED EXAMPLE

For payoffs [9, 1; 3, 7], find Row's probability p of choosing Top, Column's probability q of choosing Left, and the game value.

1. Make Column indifferent: $9p + 3(1-p) = 1p + 7(1-p)$.
2. Solve: $6p + 3 = 7 - 6p$, so $p = 1/3$.
3. Make Row indifferent: $9q + 1(1-q) = 3q + 7(1-q)$, so $q = 1/2$ and the value is 5.

Answer: Row: Top with probability 1/3. Column: Left with probability 1/2. Game value: 5.

PRACTICE 5 problems

- 1** For payoffs [8, 12; 6, 10], find a saddle point, if one exists. Give its location and value.

ANSWER _____

- 2** For payoffs [2, 14; 18, 6], let Row choose Top with probability p . Write an equation that makes Column indifferent, then find p .

ANSWER _____

- 3** For payoffs [4, 16; 20, 8], find the probability q that Column chooses Left to make Row indifferent.

ANSWER _____

- 4** For payoffs [12, 24; 28, 16], find Row's probability p of Top and Column's probability q of Left.

ANSWER _____

- 5** For payoffs [30, 34; 38, 42], use dominated strategies to find the optimal pure choices and the payoff.

ANSWER _____

APPLY IT Show your work

1

In an esports final, Team A chooses Rush or Control. Team B chooses Scout or Hold. Each entry gives Team A's score advantage: [11, 23; 27, 15]. Find Team A's probability of Rush, Team B's probability of Scout, and Team A's expected advantage.

WORK SPACE

ANSWER _____

2

Two student filmmakers compete for one festival prize. Film A chooses a Trailer or an Interview clip. Film B chooses a Comedy or Drama clip. Each entry gives Film A's judging-point advantage: [10, 24; 22, 16]. Find both optimal probabilities and the game value.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A 3 by 2 game has Row choices A, B, C and Column choices Left, Right. Payoffs to Row are [11, 31; 35, 17; 7, 9]. Explain which Row choice is dominated, then find the optimal mixed strategies and game value after removing it.

WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For payoffs [8, 12; 6, 10], find a saddle point, if one exists. Give its location and value.	Top-Left, value 8.
2	For payoffs [2, 14; 18, 6], let Row choose Top with probability p . Write an equation that makes Column indifferent, then find p .	$2p + 18(1-p) = 14p + 6(1-p)$; $p = 1/2$.
3	For payoffs [4, 16; 20, 8], find the probability q that Column chooses Left to make Row indifferent.	$q = 1/3$.
4	For payoffs [12, 24; 28, 16], find Row's probability p of Top and Column's probability q of Left.	$p = 1/2$, $q = 1/3$.
5	For payoffs [30, 34; 38, 42], use dominated strategies to find the optimal pure choices and the payoff.	Row chooses Bottom, Column chooses Left, and the payoff is 38.

PART 2 • APPLY IT

- Team A chooses Rush with probability $1/2$. Team B chooses Scout with probability $1/3$. Team A's expected advantage is 19.** Set Team B's Scout and Hold payoffs equal to find Team A's mix. Then set Team A's Rush and Control payoffs equal to find Team B's mix and substitute to get 19.
- Film A chooses Trailer with probability $3/10$ and Interview with probability $7/10$. Film B chooses Comedy with probability $2/5$ and Drama with probability $3/5$. The game value is $92/5$.** Make Film B indifferent to get Film A's Trailer probability of $3/10$. Make Film A indifferent to get Film B's Comedy probability of $2/5$, then substitute into either expected payoff.

CHALLENGE

C is dominated by A because $11 > 7$ and $31 > 9$. Row uses A with probability $9/19$, B with probability $10/19$, and C with probability 0. Column uses Left with probability $7/19$ and Right with probability $12/19$. The game value is $449/19$.

Accept equivalent fractions and decimals when exact. For mixed games, accept strategies stated as probabilities of either choice if the complementary probability is correct.