

Shortest Route Lab

Use weighted graphs to find the least-cost path.

HIGH SCHOOL • MA.912.LT.2

MA.912.LT.2 "Apply optimization and techniques from Graph Theory to solve problems."

NAME _____

DATE _____

FIND THE SHORTEST PATH

In a weighted graph, each edge has a cost, distance, or time. Add the weights on each possible route, then choose the route with the smallest total.

WORKED EXAMPLE

In an undirected graph, the edges are A-B=4, A-C=9, B-C=2, B-D=8, and C-D=3. Find the shortest path from A to D.

1. A-B-D has total weight $4 + 8 = 12$.
2. A-C-D has total weight $9 + 3 = 12$.
3. A-B-C-D has total weight $4 + 2 + 3 = 9$.
4. The smallest total is 9.

Answer: A-B-C-D, with total weight 9

PRACTICE 6 problems

- 1** In an undirected graph, the edges are P-Q=5, Q-R=6, and P-R=12. Find the shortest path from P to R.

ANSWER _____

- 2** In an undirected graph, the edges are A-B=7, A-C=10, B-C=1, B-D=5, and C-D=6. Find the shortest path from A to D.

ANSWER _____

- 3** In an undirected graph, the edges are M-N=6, N-O=7, O-P=5, M-O=15, N-P=14, and M-P=20. Find the shortest path from M to P.

ANSWER _____

- 4** In an undirected graph, the edges are W-X=10, W-Y=16, X-Y=5, X-Z=11, and Y-Z=7. Find the shortest path from W to Z.

ANSWER _____

- 5** In an undirected graph, the edges are L-M=1, L-N=10, M-N=7, M-O=10, and N-O=6. Find the shortest path from L to O.

ANSWER _____

- 6** In an undirected graph, the edges are J-K=11, J-L=6, K-L=7, K-M=10, and L-M=12. Find the shortest path from J to M.

ANSWER _____

APPLY IT Show your work

- 1** A student film crew needs to carry equipment from the Media Room (M) to the Gym (G). Paths are undirected, and the travel times are $M-A=7$ minutes, $M-C=12$ minutes, $A-C=5$ minutes, $A-G=10$ minutes, and $C-G=6$ minutes. What route takes the least time?

WORK SPACE

ANSWER _____

- 2** At a robotics tournament, a team walks from the Entrance (E) to the Pit Area (P). Paths are undirected, and the walking times are $E-R=6$ minutes, $E-C=13$ minutes, $R-C=5$ minutes, $R-P=11$ minutes, and $C-P=7$ minutes. What is the shortest route?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

In an undirected network, the edges are $S-A=5$, $S-B=7$, $A-C=6$, $B-C=1$, $A-D=12$, $C-D=5$, $C-T=13$, $D-T=6$, and $B-T=20$. Find the shortest route from S to T that must visit C. Explain how you know it is shortest.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	In an undirected graph, the edges are P-Q=5, Q-R=6, and P-R=12. Find the shortest path from P to R.	P-Q-R, with total weight 11
2	In an undirected graph, the edges are A-B=7, A-C=10, B-C=1, B-D=5, and C-D=6. Find the shortest path from A to D.	A-B-D, with total weight 12
3	In an undirected graph, the edges are M-N=6, N-O=7, O-P=5, M-O=15, N-P=14, and M-P=20. Find the shortest path from M to P.	M-N-O-P, with total weight 18
4	In an undirected graph, the edges are W-X=10, W-Y=16, X-Y=5, X-Z=11, and Y-Z=7. Find the shortest path from W to Z.	W-X-Z, with total weight 21
5	In an undirected graph, the edges are L-M=1, L-N=10, M-N=7, M-O=10, and N-O=6. Find the shortest path from L to O.	L-M-O, with total weight 11
6	In an undirected graph, the edges are J-K=11, J-L=6, K-L=7, K-M=10, and L-M=12. Find the shortest path from J to M.	J-L-M, with total weight 18

PART 2 • APPLY IT

- M-A-G, 17 minutes** Compare the possible routes from M to G. M-A-G takes $7 + 10 = 17$ minutes, which is less than the other routes.
- E-R-P, 17 minutes** Add the times for each route from E to P. E-R-P takes $6 + 11 = 17$ minutes, the smallest total.

CHALLENGE

S-B-C-D-T, 19 minutes. From S to C, S-B-C costs $7 + 1 = 8$ minutes. From C to T, C-D-T costs $5 + 6 = 11$ minutes, so the shortest required-stop route costs 19 minutes.

Accept a path only when its listed edges connect in order and its total is correct. For the challenge, require a route that includes C and an explanation comparing the shortest sections from S to C and from C to T.