

Vote Method Matchup

Use Borda count to compare ranked-choice results.

HIGH SCHOOL • MA.912.LT.3.2

MA.912.LT.3.2 "Analyze election data using election theory techniques. Explain how Arrow's Impossibility Theorem may be related to the fairness of the outcome of the election."

NAME _____

DATE _____

PART 1: PRACTICE

For three choices, Borda count gives 2 points for a first choice, 1 point for a second choice, and 0 points for a last choice. Multiply each point value by the number of voters, then add each candidate's points. Different voting methods can produce different winners.

WORKED EXAMPLE

Use Borda count: 16 voters rank $A > B > C$, 9 rank $B > C > A$, and 7 rank $C > A > B$.

1. A: $2(16) + 0(9) + 1(7) = 39$.
2. B: $1(16) + 2(9) + 0(7) = 34$.
3. C: $0(16) + 1(9) + 2(7) = 23$, so A has the most points.

Answer: A wins with 39 points.

PRACTICE 5 problems

- 1** Use Borda count: 12 rank $A > B > C$, 5 rank $B > C > A$, and 4 rank $C > A > B$.

ANSWER _____

- 2** Use Borda count: 10 rank $D > E > F$, 6 rank $E > F > D$, and 3 rank $F > D > E$.

ANSWER _____

- 3** For four choices, use 3, 2, 1, 0 points. 8 rank $G > H > I > J$, 5 rank $H > I > J > G$, and 2 rank $I > J > G > H$.

ANSWER _____

- 4** Use Borda count: 11 rank $A > B > C$, 8 rank $B > C > A$, and 6 rank $C > B > A$.

ANSWER _____

- 5** Use Borda count: 13 rank $X > Y > Z$, 4 rank $Y > Z > X$, and 2 rank $Z > X > Y$.

ANSWER _____

APPLY IT Show your work

- 1** The media club is choosing a movie-night activity. 13 students rank Game Night > Bake Sale > Movie, 10 rank Bake Sale > Movie > Game Night, and 4 rank Movie > Game Night > Bake Sale. Use Borda count.

WORK SPACE

ANSWER _____

- 2** Students are choosing a concert finale. For four choices, use 3, 2, 1, 0 points. 15 rank $R > S > T > U$, 8 rank $S > T > U > R$, and 3 rank $T > U > S > R$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

18 voters rank $A > B > C$, 17 rank $B > C > A$, and 15 rank $C > B > A$. Find the plurality winner and the Borda winner. Then explain what the different results show about fairness and Arrow's Impossibility Theorem.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Use Borda count: 12 rank $A > B > C$, 5 rank $B > C > A$, and 4 rank $C > A > B$.	A wins with 28 points.
2	Use Borda count: 10 rank $D > E > F$, 6 rank $E > F > D$, and 3 rank $F > D > E$.	D: 23, E: 22, F: 12. D wins.
3	For four choices, use 3, 2, 1, 0 points. 8 rank $G > H > I > J$, 5 rank $H > I > J > G$, and 2 rank $I > J > G > H$.	G: 26, H: 31, I: 24, J: 9. H wins.
4	Use Borda count: 11 rank $A > B > C$, 8 rank $B > C > A$, and 6 rank $C > B > A$.	B wins with 33 points.
5	Use Borda count: 13 rank $X > Y > Z$, 4 rank $Y > Z > X$, and 2 rank $Z > X > Y$.	X: 28, Y: 21, Z: 8. X wins.

PART 2 • APPLY IT

- Bake Sale wins with 33 points.** Game Night earns 30 points, Bake Sale earns 33 points, and Movie earns 18 points.
- S wins with 57 points.** The totals are R: 45, S: 57, T: 40, and U: 14.

CHALLENGE

Plurality winner: A. Borda winner: B, with A: 36, B: 67, C: 47. The results show that a voting method can change the winner. Arrow's theorem says that with three or more choices, no ranked voting method can satisfy every fairness condition.

Accept correct Borda totals even if students show multiplication in a different order. For the challenge explanation, accept statements that identify method-dependent outcomes and note that no voting system meets every fairness criterion.