LESSON MAKER PRO

# Power Behind Votes

*Use critical voters to measure influence in weighted voting.*

**HIGH SCHOOL · MA.912.LT.3.3**

**MA.912.LT.3.3** "Decide voting power within a group using weighted voting techniques. Provide real-world examples of weighted voting and its pros and cons."

NAME _____ DATE _____

## FINDING VOTING POWER

In a system written [q: weights], q is the quota needed to win. A voter is critical in a winning coalition if removing that voter makes the coalition lose. Count each voter's critical appearances, then divide each count by the total number of critical appearances to find normalized Banzhaf power. Weighted voting can represent different group sizes or roles, but it can also give some voters much more influence than others.

**WORKED EXAMPLE**

Find the normalized Banzhaf power of each voter in [41: 24, 15, 9].

1. The only winning coalition is ABC because 24 + 15 + 9 = 48, and every pair totals less than 41.
2. In ABC, removing A leaves 24, removing B leaves 33, and removing C leaves 39. Each total loses.
3. The critical counts are A = 1, B = 1, and C = 1, for 3 critical appearances total.
4. Divide each count by 3.

**Answer: A = 1/3, B = 1/3, C = 1/3**

## PRACTICE 6 problems

**1** In [11: 7, 4, 3], list all winning coalitions.

**ANSWER** _____

**2** In [12: 7, 3, 2], which voters are critical in coalition ABC?

**ANSWER** _____

**3** Find the raw Banzhaf counts for A, B, and C in [10: 6, 4, 1].

**ANSWER** _____

**4** Find voter B's normalized Banzhaf power in [7: 4, 3, 1].

**ANSWER** _____

**5** In [8: 8, 2, 1], which voter is a dictator?

**ANSWER** _____

**6** In [18: 12, 7, 4, 3], does any voter have zero Banzhaf power? Give the raw Banzhaf counts.

**ANSWER** _____

## APPLY IT Show your work

**1** A student media board votes on a video project using [13: 7, 5, 4, 2]. Find voter C's normalized Banzhaf power. Then choose the benefit of weighted voting: A. It can reflect different group sizes or responsibilities. B. It guarantees equal voting power for every voter.

WORK SPACE

**ANSWER** _____

**2** A youth advisory committee votes on a school event using [16: 8, 5, 4, 3]. Find voter B's normalized Banzhaf power. Then choose the concern about weighted voting: A. A small-weight voter can have less influence than a large-weight voter. B. Every voter must have the same influence.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

In [16: 7, 5, 4, 3, 2], find the normalized Banzhaf power of all five voters. Which voter has the greatest voting power? Explain using the critical counts.

WORK SPACE AND ANSWER

ANSWER KEY

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## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | In [11: 7, 4, 3], list all winning coalitions. | **AB and ABC** |
| 2 | In [12: 7, 3, 2], which voters are critical in coalition ABC? | **A, B, and C** |
| 3 | Find the raw Banzhaf counts for A, B, and C in [10: 6, 4, 1]. | **A = 2, B = 2, C = 0** |
| 4 | Find voter B's normalized Banzhaf power in [7: 4, 3, 1]. | **1/2** |
| 5 | In [8: 8, 2, 1], which voter is a dictator? | **A** |
| 6 | In [18: 12, 7, 4, 3], does any voter have zero Banzhaf power? Give the raw Banzhaf counts. | **No. A = 5, B = 3, C = 1, D = 1.** |

## PART 2 · APPLY IT

1. **C = 1/5; A** The raw Banzhaf counts are A = 4, B = 2, C = 2, and D = 2, for 10 total critical appearances. Voter C has 2/10 = 1/5 of the power, and choice A states a benefit.
2. **B = 3/8; A** The raw Banzhaf counts are A = 3, B = 3, C = 1, and D = 1, for 8 total critical appearances. Voter B has 3/8 of the power, and choice A describes a possible concern.

## CHALLENGE

**A = 1/3, B = 2/9, C = 2/9, D = 1/9, E = 1/9. A has the greatest power because A is critical in 6 coalitions, while B and C are each critical in 4 coalitions and D and E are each critical in 2 coalitions.**

Accept equivalent fractions, such as 2/10 for 1/5, when the fraction represents normalized power. Accept coalition names in any order, provided the listed voters and critical counts are correct.