

Logic Reality Check

Test claims, test arguments, and find the exception.

HIGH SCHOOL • MA.912.LT.4.10

MA.912.LT.4.10 "Judge the validity of arguments and give counterexamples to disprove statements."

NAME _____

DATE _____

TEST THE LOGIC

An argument is valid when the premises cannot be true while the conclusion is false. To disprove a statement that says something is always true, find one counterexample that makes the statement fail.

WORKED EXAMPLE

Is this statement true? Every integer greater than 8 is even.

1. Test the integer 9.
2. 9 is greater than 8.
3. 9 is odd, not even.

Answer: False. A counterexample is 9.

PRACTICE 6 problems

- 1** Judge this argument: If an integer is divisible by 4, then it is even. 18 is even. Therefore, 18 is divisible by 4.

ANSWER _____

- 2** Is this statement true? For every integer x , if x^2 is even, then x is even.

ANSWER _____

- 3** Judge this argument: For any real x , if $x > 5$, then $x^2 > 25$. 3^2 is not greater than 25. Therefore, 3 is not greater than 5.

ANSWER _____

- 4** Is this statement true? All prime numbers are odd. If it is false, give a counterexample.

ANSWER _____

- 5** Judge this argument: If a student joins the debate club, then the student attends the first meeting. Maya attended the first meeting. Therefore, Maya joined the debate club.

ANSWER _____

- 6** Is this statement true? For every integer x , if x is odd, then x^2 is odd.

ANSWER _____

APPLY IT Show your work

- 1** An esports tournament uses this rule: If a team wins every match, then it earns a playoff spot. Team Nova earned a playoff spot. A commentator concludes that Team Nova won every match. Is the commentator's argument valid? Give a counterexample if it is invalid.

WORK SPACE

ANSWER _____

- 2** A career teacher says, "If a student uploads a portfolio by the deadline, then the student receives feedback." Eli did not receive feedback. The teacher concludes that Eli did not upload a portfolio by the deadline. Is this argument valid?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A student claims, "For every integer x , if x^2 is divisible by 12, then x is divisible by 12." Is the claim true? Explain your reasoning with a counterexample if needed.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Judge this argument: If an integer is divisible by 4, then it is even. 18 is even. Therefore, 18 is divisible by 4.	Invalid. 18 is even but is not divisible by 4.
2	Is this statement true? For every integer x , if x^2 is even, then x is even.	True.
3	Judge this argument: For any real x , if $x > 5$, then $x^2 > 25$. 3^2 is not greater than 25. Therefore, 3 is not greater than 5.	Valid.
4	Is this statement true? All prime numbers are odd. If it is false, give a counterexample.	False. A counterexample is 2.
5	Judge this argument: If a student joins the debate club, then the student attends the first meeting. Maya attended the first meeting. Therefore, Maya joined the debate club.	Invalid. Maya could have attended the meeting as a guest without joining the club.
6	Is this statement true? For every integer x , if x is odd, then x^2 is odd.	True.

PART 2 • APPLY IT

1. **Invalid. Team Nova could have lost one match and still earned a playoff spot.** The rule says winning every match is enough to earn a spot, but it does not say it is the only way. A team could earn a spot with a loss.
2. **Valid.** This uses the contrapositive form. If uploading leads to feedback, then no feedback means the portfolio was not uploaded by the deadline.

CHALLENGE

False. Let $x = 6$. Then $6^2 = 36$, and 36 is divisible by 12, but 6 is not divisible by 12.

Accept any valid counterexample that makes the premises true and conclusion false, or that makes a universal statement fail. For true statements and valid arguments, accept a correct statement that no counterexample exists.