

Logic Flip Lab

Turn statements around, negate them, and test their meaning.

HIGH SCHOOL • MA.912.LT.4.3

MA.912.LT.4.3 "Identify and accurately interpret "if...then," "if and only if," "all" and "not" statements. Find the converse, inverse and contrapositive of a statement."

NAME _____

DATE _____

CONDITIONAL STATEMENT MOVES

For a conditional "If P, then Q," the converse is "If Q, then P," the inverse is "If not P, then not Q," and the contrapositive is "If not Q, then not P." A conditional and its contrapositive always have the same truth value, and the converse and inverse always have the same truth value. "P if and only if Q" means both directions are true, while "not all" means there is at least one exception.

WORKED EXAMPLE

For the statement "If an integer is a multiple of 12, then it is even," write the converse, inverse, and contrapositive.

1. Let P = "the integer is a multiple of 12."
2. Let Q = "the integer is even."
3. Switch P and Q to write the converse.
4. Negate both parts and switch them to write the contrapositive.

Answer: Converse: If an integer is even, then it is a multiple of 12. Inverse: If an integer is not a multiple of 12, then it is not even. Contrapositive: If an integer is not even, then it is not a multiple of 12.

PRACTICE 6 problems

- 1** Write the converse of: If x is divisible by 4, then x is even.

ANSWER _____

- 2** Write the inverse of: If $x > 7$, then $x > 2$.

ANSWER _____

- 3** Write the contrapositive of: If a number is divisible by 10, then it ends in 0.

ANSWER _____

- 4** Write the two conditional statements meant by: A triangle is isosceles if and only if it has at least two congruent sides.

ANSWER _____

- 5** Write the negation of: All multiples of 6 are even.

ANSWER _____

- 6** Interpret this statement in your own words: It is not true that every book in the library is checked out.

ANSWER _____

APPLY IT Show your work

- 1** A gaming club uses this access rule: "If a member completes the online safety quiz, then the member can borrow a virtual-reality headset." Write the converse, inverse, and contrapositive. Do not assume that any new statement is a club rule.

WORK SPACE

ANSWER _____

- 2** A student council announcement says, "All students who turn in a signed permission form by Friday may attend the weekend concert trip." Later, the adviser says, "It is not true that all students who signed up turned in the form by Friday." What must be true?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let P be " x is divisible by 3" and Q be " x^2 is divisible by 3," where x is an integer. For the conditional "If P , then Q ," write its converse, inverse, and contrapositive. Then state which of the four statements are true for every integer x , and explain.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write the converse of: If x is divisible by 4, then x is even.	If x is even, then x is divisible by 4.
2	Write the inverse of: If $x > 7$, then $x > 2$.	If $x \leq 7$, then $x \leq 2$.
3	Write the contrapositive of: If a number is divisible by 10, then it ends in 0.	If a number does not end in 0, then it is not divisible by 10.
4	Write the two conditional statements meant by: A triangle is isosceles if and only if it has at least two congruent sides.	If a triangle is isosceles, then it has at least two congruent sides. If a triangle has at least two congruent sides, then it is isosceles.
5	Write the negation of: All multiples of 6 are even.	At least one multiple of 6 is not even.
6	Interpret this statement in your own words: It is not true that every book in the library is checked out.	At least one book in the library is not checked out.

PART 2 • APPLY IT

- Converse:** If a member can borrow a virtual-reality headset, then the member completed the online safety quiz. **Inverse:** If a member does not complete the online safety quiz, then the member cannot borrow a virtual-reality headset. **Contrapositive:** If a member cannot borrow a virtual-reality headset, then the member did not complete the online safety quiz. Let P be completing the quiz and Q be being able to borrow the headset. Switch P and Q for the converse, negate both without switching for the inverse, and negate both while switching for the contrapositive.
- At least one student who signed up did not turn in a signed permission form by Friday.** "It is not true that all" means there is at least one exception. The exception is a student who signed up but did not turn in the form by Friday.

CHALLENGE

Converse: If x^2 is divisible by 3, then x is divisible by 3. **Inverse:** If x is not divisible by 3, then x^2 is not divisible by 3. **Contrapositive:** If x^2 is not divisible by 3, then x is not divisible by 3. All four statements are true for every integer x . If x is divisible by 3, then $x = 3k$ for some integer k , so $x^2 = 9k^2$ and is divisible by 3. If x is not divisible by 3, its remainder is 1 or 2, and its square has remainder 1, so x^2 is not divisible by 3.

Accept equivalent wording for negations, such as " x is not greater than 7" for " $x \leq 7$." For the challenge, accept a correct divisibility argument showing that an integer not divisible by 3 cannot have a square divisible by 3.