LESSON MAKER PRO

Proof Power Check

Test arguments and build clear proofs.

HIGH SCHOOL · MA.912.LT.4.6

MA.912.LT.4.6 "Apply methods of direct and indirect proof and determine whether a logical argument is valid."

NAME _____ DATE _____

VALID PATHS

An argument is valid when true premises force the conclusion to be true, and one case with true premises and a false conclusion makes it invalid. In a direct proof, start with the given information and use definitions or known facts to reach the conclusion. In an indirect proof, assume the conclusion is false and show that this assumption creates a contradiction.

WORKED EXAMPLE

Decide whether this argument is valid: If p, then q. If q, then r. p. Therefore r.

1. Start with p, because p is a premise.
2. Use if p, then q to conclude q.
3. Use if q, then r to conclude r.
4. The conclusion follows from the premises, so the argument is valid.

Answer: Valid.

PRACTICE 6 problems

1 Is this argument valid? If a whole number is divisible by 10, then it is even. 30 is divisible by 10. Therefore, 30 is even.

ANSWER _____

2 Is this argument valid? If a whole number is a multiple of 4, then it is even. 18 is even. Therefore, 18 is a multiple of 4.

ANSWER _____

3 Is this argument valid? If a whole number n is divisible by 12, then n is divisible by 3. n is not divisible by 3. Therefore, n is not divisible by 12.

ANSWER _____

4 Write a direct proof: If x is a multiple of 6, then x is a multiple of 3.

ANSWER _____

5 Use an indirect proof to prove: If x is an integer and x^2 is odd, then x is odd.

ANSWER _____

6 Is this argument valid? If a whole number x is divisible by 5, then x ends in 0 or 5. x does not end in 0 or 5. Therefore, x is not divisible by 5.

ANSWER _____

APPLY IT Show your work

1 A college art program says, "If a student submits a portfolio, then the student's application is reviewed." Nia's application was reviewed. The program concludes that Nia submitted a portfolio. Is the conclusion logically valid? Explain.

WORK SPACE

ANSWER _____

2 The drama club puts 8 buttons in each full giveaway bag. After packing, the club counts 58 buttons, and all buttons were placed in bags. Use an indirect proof to show that at least one bag is not full.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let x be a whole number. Decide whether this argument is valid, and explain your reasoning: If x is divisible by 4, then x is even. If x is even, then x^2 is divisible by 4. x^2 is not divisible by 4. Therefore, x is not divisible by 4.

WORK SPACE AND ANSWER

ANSWER KEY

Proof Power Check

HIGH SCHOOL · MA.912.LT.4.6

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Is this argument valid? If a whole number is divisible by 10, then it is even. 30 is divisible by 10. Therefore, 30 is even.	**Valid.**
2	Is this argument valid? If a whole number is a multiple of 4, then it is even. 18 is even. Therefore, 18 is a multiple of 4.	**Invalid. 18 is even but is not a multiple of 4.**
3	Is this argument valid? If a whole number n is divisible by 12, then n is divisible by 3. n is not divisible by 3. Therefore, n is not divisible by 12.	**Valid.**
4	Write a direct proof: If x is a multiple of 6, then x is a multiple of 3.	**Let x = 6k for some integer k. Then x = 3(2k), and 2k is an integer. Therefore, x is a multiple of 3.**
5	Use an indirect proof to prove: If x is an integer and x^2 is odd, then x is odd.	**Assume x is not odd. Then x = 2k for some integer k, so $x^2 = 4k^2 = 2(2k^2)$, which is even. This contradicts x^2 being odd. Therefore, x is odd.**
6	Is this argument valid? If a whole number x is divisible by 5, then x ends in 0 or 5. x does not end in 0 or 5. Therefore, x is not divisible by 5.	**Valid.**

PART 2 · APPLY IT

1. **Invalid.** Nia's application could have been reviewed even if she did not submit a portfolio. This is affirming the consequent. A counterexample occurs when Nia does not submit a portfolio, but her application is reviewed for another reason.
2. **At least one bag is not full.** Assume every bag is full. Then 58 would equal 8b for a whole number b, so 58 would be divisible by 8. But 58 = 56 + 2, so it is not divisible by 8. Therefore, at least one bag is not full. Assume all bags are full, so the total must be a multiple of 8. Since 58 is not a multiple of 8, the assumption creates a contradiction.

CHALLENGE

Valid. If x were divisible by 4, then x would be even, and x^2 would be divisible by 4. Since x^2 is not divisible by 4, x cannot be divisible by 4.

Accept equivalent proof wording and any correct integer variable in the direct and indirect proofs. For invalid arguments, accept any explanation that gives a case where the premises are true and the conclusion is false.