

Proof Pattern Lab

Use logic to build a claim that lasts.

HIGH SCHOOL • MA.912.LT.4.7

MA.912.LT.4.7 "Identify and give examples of undefined terms; axioms; theorems; proofs, including proofs using mathematical induction; and inductive and deductive reasoning."

NAME _____

DATE _____

PROOF TOOLS

Undefined terms are basic starting ideas. An axiom is accepted as a starting rule, while a theorem is proved from axioms and earlier results. Induction uses a base case and then shows that $P(k)$ leads to $P(k+1)$.

WORKED EXAMPLE

Prove by induction: $10+20+\dots+10n = 5n(n+1)$ for $n \geq 1$.

1. Base case: $n=1$ gives $10=5(1)(2)$.
2. Assume $10+20+\dots+10k=5k(k+1)$.
3. Add $10(k+1)$: $5k(k+1)+10(k+1)=5(k+1)(k+2)$.
4. So $P(k+1)$ is true, and the claim holds for all $n \geq 1$.

Answer: $10+20+\dots+10n = 5n(n+1)$ for all $n \geq 1$.

PRACTICE 5 problems

- 1** Name the undefined terms in geometry.

ANSWER _____

- 2** Classify this statement: A unique line passes through a pair of distinct points.

ANSWER _____

- 3** A result proved from axioms and earlier results using deductive reasoning is a ____.

ANSWER _____

- 4** A student checks several terms of a pattern, sees that each is odd, and predicts all terms will be odd. What type of reasoning is this?

ANSWER _____

- 5** Assume $8+16+\dots+8k=4k(k+1)$. Write the statement you must prove for $P(k+1)$.

ANSWER _____

APPLY IT Show your work

- 1** A gaming app gives 9 bonus tokens on day one, 18 on day two, and $9n$ on day n . Write a formula for the total after n days. State the base-case total.

WORK SPACE

ANSWER _____

- 2** A video creator posts 11 clips in week one, 22 in week two, and $11n$ in week n . Write the total through n weeks. If $P(k)$ is assumed, what amount is added for $P(k+1)$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Prove by induction: $7+49+\dots+7^n = \frac{7^{n+1}-7}{6}$ for $n \geq 1$.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Name the undefined terms in geometry.	point, line, plane
2	Classify this statement: A unique line passes through a pair of distinct points.	axiom or postulate
3	A result proved from axioms and earlier results using deductive reasoning is a ____.	theorem
4	A student checks several terms of a pattern, sees that each is odd, and predicts all terms will be odd. What type of reasoning is this?	inductive reasoning
5	Assume $8+16+\dots+8k=4k(k+1)$. Write the statement you must prove for $P(k+1)$.	$8+16+\dots+8k+8(k+1)=4(k+1)(k+2)$

PART 2 · APPLY IT

- Total = $9n(n+1)/2$. Base case: $n=1$, total=9.** Add $9+18+\dots+9n$, then factor out 9. Check the first day separately for the base case.
- Total = $11n(n+1)/2$. Add $11(k+1)$.** The weekly amounts are multiples of 11. The next weekly amount after week k is $11(k+1)$.

CHALLENGE

Base: $n=1$ gives $7=(7^2-7)/6$. Assume $7+49+\dots+7^k=(7^{k+1}-7)/6$. Then adding 7^{k+1} gives $(7^{k+1}-7)/6+7^{k+1}=(7^{k+2}-7)/6$, so the claim holds.

Accept "postulate" for axiom. Accept equivalent algebraic forms for induction statements and totals.