LESSON MAKER PRO

Proof Pattern Lab

Use logic to build a claim that lasts.

HIGH SCHOOL · MA.912.LT.4.7

MA.912.LT.4.7 "Identify and give examples of undefined terms; axioms; theorems; proofs, including proofs using mathematical induction; and inductive and deductive reasoning."

NAME _____ DATE _____

PROOF TOOLS

Undefined terms are basic starting ideas. An axiom is accepted as a starting rule, while a theorem is proved from axioms and earlier results. Induction uses a base case and then shows that P(k) leads to P(k+1).

WORKED EXAMPLE

Prove by induction: $10+20+\ldots+10n = 5n(n+1)$ for $n>=1$.

1. Base case: n=1 gives 10=5(1)(2).
2. Assume 10+20+...+10k=5k(k+1).
3. Add 10(k+1): 5k(k+1)+10(k+1)=5(k+1)(k+2).
4. So P(k+1) is true, and the claim holds for all n>=1.

Answer: 10+20+...+10n = 5n(n+1) for all n>=1.

PRACTICE 5 problems

1 Name the undefined terms in geometry.

ANSWER _____

2 Classify this statement: A unique line passes through a pair of distinct points.

ANSWER _____

3 A result proved from axioms and earlier results using deductive reasoning is a ___.

ANSWER _____

4 A student checks several terms of a pattern, sees that each is odd, and predicts all terms will be odd. What type of reasoning is this?

ANSWER _____

5 Assume 8+16+...+8k=4k(k+1). Write the statement you must prove for P(k+1).

ANSWER _____

APPLY IT Show your work

1 A gaming app gives 9 bonus tokens on day one, 18 on day two, and 9n on day n. Write a formula for the total after n days. State the base-case total.

WORK SPACE

ANSWER _____

2 A video creator posts 11 clips in week one, 22 in week two, and 11n in week n. Write the total through n weeks. If P(k) is assumed, what amount is added for P(k+1)?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Prove by induction: $7+49+\ldots+7^n=(7^{(n+1)}-7)/6$ for $n>=1$.

WORK SPACE AND ANSWER

Proof Pattern Lab

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Name the undefined terms in geometry.	**point, line, plane**
2	Classify this statement: A unique line passes through a pair of distinct points.	**axiom or postulate**
3	A result proved from axioms and earlier results using deductive reasoning is a ___.	**theorem**
4	A student checks several terms of a pattern, sees that each is odd, and predicts all terms will be odd. What type of reasoning is this?	**inductive reasoning**
5	Assume 8+16+...+8k=4k(k+1). Write the statement you must prove for P(k+1).	**8+16+...+8k+8(k+1)=4(k+1)(k+2)**

PART 2 · APPLY IT

1. **Total = 9n(n+1)/2. Base case: n=1, total=9.** Add 9+18+...+9n, then factor out 9. Check the first day separately for the base case.
2. **Total = 11n(n+1)/2. Add 11(k+1).** The weekly amounts are multiples of 11. The next weekly amount after week k is 11(k+1).

CHALLENGE

Base: n=1 gives $7=(7^2-7)/6$. Assume $7+49+...+7^k=(7^{(k+1)}-7)/6$. Then adding $7^{(k+1)}$ gives $(7^{(k+1)}-7)/6+7^{(k+1)}=(7^{(k+2)}-7)/6$, so the claim holds.

Accept “postulate” for axiom. Accept equivalent algebraic forms for induction statements and totals.