LESSON MAKER PRO

Proof Pivot

Build direct proofs and contradiction proofs.

HIGH SCHOOL · MA.912.LT.4.8

MA.912.LT.4.8 "Construct proofs, including proofs by contradiction."

NAME ______________________ DATE ____________

CONTRADICTION STEPS

To prove a statement by contradiction, assume its opposite is true. Use definitions and algebra until the assumption leads to an impossible result, then reject the assumption.

WORKED EXAMPLE

Prove by contradiction: No integer x has $x^2 = 90$.

1. Assume an integer x has $x^2 = 90$.
2. A number of the form x^2 must be a perfect square.
3. $9^2 = 81 < 90 < 100 = 10^2$, so 90 is not a perfect square.
4. This contradicts $x^2 = 90$.

Answer: No integer x has $x^2 = 90$.

PRACTICE 5 problems

1. Prove by contradiction: No integer x has $x^2 = 3$.

   ANSWER ____________________

2. Prove: If x is odd, then x^2 is odd.

   ANSWER ____________________

3. Prove by contradiction: If x^2 is even, then x is even.

   ANSWER ____________________

4. Prove: The sum of two odd integers is even.

   ANSWER ____________________

5. Prove by contradiction: There is no greatest even integer.

   ANSWER ____________________

APPLY IT Show your work

1. A yearbook editor wants a square photo mosaic with exactly 45 photos, using the same number of photos in each row and column. Prove this plan is impossible.

WORK SPACE

ANSWER ______________________

2. An esports club has 13 members. A schedule claims that each member will play exactly 7 matches against other club members. Prove the schedule is impossible.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

Use contradiction to prove that sqrt(2) is irrational.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Prove by contradiction: No integer x has x^2 = 3.	**Assume x^2 = 3 for an integer x. Since 1^2 < 3 < 2^2, 3 is not a perfect square. This is a contradiction, so no such integer x exists.**
2	Prove: If x is odd, then x^2 is odd.	**Let x = 2k + 1. Then x^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1, so x^2 is odd.**
3	Prove by contradiction: If x^2 is even, then x is even.	**Assume x^2 is even but x is odd. Then x = 2k + 1, so x^2 = 2(2k^2 + 2k) + 1, which is odd. This contradicts x^2 being even, so x is even.**
4	Prove: The sum of two odd integers is even.	**Let the integers be 2a + 1 and 2b + 1. Their sum is 2a + 2b + 2 = 2(a + b + 1), so it is even.**
5	Prove by contradiction: There is no greatest even integer.	**Assume N is the greatest even integer. Then N + 2 is even and greater than N, a contradiction. Therefore, there is no greatest even integer.**

PART 2 · APPLY IT

1. **The plan is impossible.** If there are x photos in each row and column, then x^2 = 45. Since 6^2 < 45 < 7^2, 45 is not a perfect square.
2. **The schedule is impossible.** The schedule would count 13 x 7 = 91 player-match entries. Each match gives exactly 2 entries, so the total must be even, but 91 is odd.

CHALLENGE

Assume sqrt(2) = a/b for integers a and b with no common factor. Then a^2 = 2b^2, so a^2 is even and a is even. Let a = 2k. Then 4k^2 = 2b^2, so b^2 = 2k^2 and b is even. Both a and b are even, contradicting that the fraction was in lowest terms. Therefore sqrt(2) is irrational.

Accept equivalent variable names and equivalent algebraic forms. For contradiction proofs, students must state an assumption opposite the claim and identify the contradiction.