LESSON MAKER PRO

Logic Checkpoint

Test arguments by looking for counterexamples.

HIGH SCHOOL · MA.912.LT.4

MA.912.LT.4 "Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof."

NAME _____ DATE _____

VALID OR INVALID?

A conditional p -> q is false only when p is true and q is false. An argument is valid when no assignment of truth values makes every premise true and the conclusion false. To test an argument, try to create that one counterexample.

WORKED EXAMPLE

Test the argument: Premises: r -> s and r. Conclusion: s.

1. To test validity, make the conclusion s false.
2. The premise r requires r to be true.
3. When r is true and s is false, r -> s is false.
4. The premises cannot all be true while the conclusion is false.

Answer: Valid.

PRACTICE 6 problems

1 Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and q. Conclusion: p.

ANSWER _____

2 Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and not q. Conclusion: not p.

ANSWER _____

3 Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p or q and not p. Conclusion: q.

ANSWER _____

4 Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q, q -> r, and p. Conclusion: r.

ANSWER _____

5 Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and not p. Conclusion: not q.

ANSWER _____

6 Test the argument. State valid or invalid. If invalid, give a counterexample. Premise: p and q. Conclusion: p.

ANSWER _____

APPLY IT Show your work

1 At the school esports club, the rule is: If a player registers for the tournament, then the player receives a team email. Jordan received a team email and concludes, "I registered for the tournament." Is Jordan's conclusion logically valid? Explain using p and q.

WORK SPACE

ANSWER _____

2 The drama club says: If a student volunteers at rehearsal, then the student earns service credit. If a student earns service credit, then the student is listed in the program. Mia volunteered at rehearsal. Can you validly conclude that Mia is listed in the program? Explain.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Test the argument without making a full truth table. Explain your reasoning. Premises: p -> q, q -> r, r -> s, and not s. Conclusion: not p.

WORK SPACE AND ANSWER

Logic Checkpoint

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and q. Conclusion: p.	**Invalid. Counterexample: p = false, q = true.**
2	Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and not q. Conclusion: not p.	**Valid.**
3	Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p or q and not p. Conclusion: q.	**Valid.**
4	Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q, q -> r, and p. Conclusion: r.	**Valid.**
5	Test the argument. State valid or invalid. If invalid, give a counterexample. Premises: p -> q and not p. Conclusion: not q.	**Invalid. Counterexample: p = false, q = true.**
6	Test the argument. State valid or invalid. If invalid, give a counterexample. Premise: p and q. Conclusion: p.	**Valid.**

PART 2 · APPLY IT

1. **Invalid. Let p mean "Jordan registered" and q mean "Jordan received a team email." If p is false and q is true, the rule and the premise are true, but Jordan's conclusion is false.** This is the form p -> q, q, therefore p. A true team email does not prove that registration happened.
2. **Valid.** Mia volunteered, so she earns service credit. Because she earns service credit, she is listed in the program. This chains two conditionals: volunteer -> credit and credit -> listed. Since Mia volunteered, the conclusion follows.

CHALLENGE

Valid. If p were true, then q, r, and s would all be true. That contradicts the premise not s, so p must be false.

Accept T and F in place of true and false. For invalid arguments, accept any truth-value assignment that makes all premises true and the conclusion false.