

Relation Switch Lab

Test functions, reverse relations, and spot bijections.

HIGH SCHOOL • MA.912.LT.5.2

MA.912.LT.5.2 "Given a relation on two sets, determine whether the relation is a function, determine the inverse of the relation if it exists and identify if the relation is bijective."

NAME _____

DATE _____

FUNCTION AND INVERSE CHECK

A relation from A to B is a function when each input in A has exactly one output. Its inverse switches each ordered pair, and the inverse is a function exactly when no two inputs share an output. A relation is bijective when it is a function, every element of B is used, and no output repeats.

WORKED EXAMPLE

Let $A=\{12,15\}$, $B=\{r,s\}$, and $R=\{(12,r),(15,s)\}$. Decide whether R is bijective and find R^{-1} .

1. Each input, 12 and 15, has one output.
2. Each element of B, r and s, is used once.
3. Switch the coordinates in each ordered pair.

Answer: R is bijective. $R^{-1}=\{(r,12),(s,15)\}$.

PRACTICE 6 problems

- 1** Let $R=\{(-3,4),(0,1),(2,4)\}$. Is R a function? Write R^{-1} . Is R^{-1} a function?

ANSWER _____

- 2** Let $R=\{(1,2),(1,5),(3,4)\}$. Is R a function? State the ordered-pair fact that proves your answer.

ANSWER _____

- 3** Let $A=\{2,5,7\}$, $B=\{a,b,c\}$, and $R=\{(2,b),(5,a),(7,c)\}$. Is R bijective? Write R^{-1} .

ANSWER _____

- 4** Let R be a relation from $X=\{-2,0,2,4\}$ to $Y=\{1,3,5\}$: $R=\{(-2,1),(0,3),(2,5),(4,5)\}$. Is R a function? Is it onto Y? Is it bijective? Is R^{-1} a function?

ANSWER _____

- 5** Let R be a relation from $A=\{6,7,9\}$ to $B=\{6,9,10\}$: $R=\{(6,9),(7,10),(9,6)\}$. Is R bijective? Write R^{-1} .

ANSWER _____

- 6** The function f maps $A=\{0,1,2,3\}$ to $B=\{-1,1,3,5\}$ by $f(x)=2x-1$. Is f bijective? Write $f^{-1}(x)$.

ANSWER _____

APPLY IT Show your work

- 1** Four students each submit one poster design. The relation from students $A=\{\text{Inez, Kai, Lila, Omar}\}$ to poster codes $B=\{P1, P2, P3, P4\}$ is $\{(\text{Inez}, P3), (\text{Kai}, P1), (\text{Lila}, P4), (\text{Omar}, P2)\}$. Is the relation a function? Is it bijective? Write the inverse relation.

WORK SPACE

ANSWER _____

- 2** For a gaming tournament, players are assigned practice rooms. The relation from players $A=\{\text{Maya, Noah, Priya, Sam}\}$ to rooms $B=\{\text{Blue, Gold, Green}\}$ is $\{(\text{Maya}, \text{Blue}), (\text{Noah}, \text{Gold}), (\text{Priya}, \text{Green}), (\text{Sam}, \text{Blue})\}$. Is the relation a function? Is it bijective? Write the inverse relation, then decide whether the inverse is a function.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let $A=\{-1, 0, 1, 2\}$ and $B=\{2, 4, 6, 8\}$. The relation R from A to B is $\{(-1, a), (0, 4), (1, 6), (2, b)\}$, where a and b are elements of B and $a < b$. Find a and b so that R is bijective. Then write R^{-1} and explain why your values work.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Let $R=\{(-3,4),(0,1),(2,4)\}$. Is R a function? Write R^{-1} . Is R^{-1} a function?	R is a function. $R^{-1}=\{(4,-3),(1,0),(4,2)\}$. R^{-1} is not a function because 4 maps to both -3 and 2.
2	Let $R=\{(1,2),(1,5),(3,4)\}$. Is R a function? State the ordered-pair fact that proves your answer.	No. The input 1 maps to both 2 and 5.
3	Let $A=\{2,5,7\}$, $B=\{a,b,c\}$, and $R=\{(2,b),(5,a),(7,c)\}$. Is R bijective? Write R^{-1} .	R is bijective. $R^{-1}=\{(b,2),(a,5),(c,7)\}$.
4	Let R be a relation from $X=\{-2,0,2,4\}$ to $Y=\{1,3,5\}$: $R=\{(-2,1),(0,3),(2,5),(4,5)\}$. Is R a function? Is it onto Y ? Is it bijective? Is R^{-1} a function?	R is a function and is onto Y. It is not bijective. R^{-1} is not a function because 5 maps to both 2 and 4.
5	Let R be a relation from $A=\{6,7,9\}$ to $B=\{6,9,10\}$: $R=\{(6,9),(7,10),(9,6)\}$. Is R bijective? Write R^{-1} .	R is bijective. $R^{-1}=\{(9,6),(10,7),(6,9)\}$.
6	The function f maps $A=\{0,1,2,3\}$ to $B=\{-1,1,3,5\}$ by $f(x)=2x-1$. Is f bijective? Write $f^{-1}(x)$.	f is bijective. $f^{-1}(x)=(x+1)/2$.

PART 2 • APPLY IT

- It is a function and is bijective. The inverse relation is $\{(P3,Inez),(P1,Kai),(P4,Lila),(P2,Omar)\}$.** Each student has one poster code, and every code is used exactly once. Reverse each student-code pair to form the inverse.
- It is a function but not bijective. The inverse relation is $\{(Blue,Maya),(Gold,Noah),(Green,Priya),(Blue,Sam)\}$. The inverse is not a function.** Every player has one room, so the relation is a function, and all three rooms are used. Blue is assigned to two players, so the relation is not one-to-one and its inverse is not a function.

CHALLENGE

$a=2$ and $b=8$. $R^{-1}=\{(2,-1),(4,0),(6,1),(8,2)\}$. The outputs are 2,4,6,8, so every element of B is used once and no output repeats.

Accept inverse relations in any order. A reversed relation may be written even when it is not a function, but call it an inverse function only when each reversed input has exactly one output.