LESSON MAKER PRO

Relation Switch Lab

Test functions, reverse relations, and spot bijections.

HIGH SCHOOL · MA.912.LT.5.2

MA.912.LT.5.2 "Given a relation on two sets, determine whether the relation is a function, determine the inverse of the relation if it exists and identify if the relation is bijective."

NAME _____ DATE _____

FUNCTION AND INVERSE CHECK

A relation from A to B is a function when each input in A has exactly one output. Its inverse switches each ordered pair, and the inverse is a function exactly when no two inputs share an output. A relation is bijective when it is a function, every element of B is used, and no output repeats.

WORKED EXAMPLE

Let A={12,15}, B={r,s}, and R={(12,r),(15,s)}. Decide whether R is bijective and find R^{-1}.

1. Each input, 12 and 15, has one output.
2. Each element of B, r and s, is used once.
3. Switch the coordinates in each ordered pair.

Answer: R is bijective. R^{-1}={(r,12),(s,15)}.

PRACTICE 6 problems

1 Let R={(-3,4),(0,1),(2,4)}. Is R a function? Write R^{-1}. Is R^{-1} a function?

ANSWER _____

2 Let R={(1,2),(1,5),(3,4)}. Is R a function? State the ordered-pair fact that proves your answer.

ANSWER _____

3 Let A={2,5,7}, B={a,b,c}, and R={(2,b),(5,a),(7,c)}. Is R bijective? Write R^{-1}.

ANSWER _____

4 Let R be a relation from X={-2,0,2,4} to Y={1,3,5}: R={(-2,1),(0,3),(2,5),(4,5)}. Is R a function? Is it onto Y? Is it bijective? Is R^{-1} a function?

ANSWER _____

5 Let R be a relation from A={6,7,9} to B={6,9,10}: R={(6,9),(7,10),(9,6)}. Is R bijective? Write R^{-1}.

ANSWER _____

6 The function f maps A={0,1,2,3} to B={-1,1,3,5} by f(x)=2x-1. Is f bijective? Write $f^{-1}(x)$.

ANSWER _____

APPLY IT Show your work

1 Four students each submit one poster design. The relation from students A={Inez,Kai,Lila,Omar} to poster codes B={P1,P2,P3,P4} is {(Inez,P3),(Kai,P1),(Lila,P4),(Omar,P2)}. Is the relation a function? Is it bijective? Write the inverse relation.

WORK SPACE

ANSWER _____

2 For a gaming tournament, players are assigned practice rooms. The relation from players A={Maya,Noah,Priya,Sam} to rooms B={Blue,Gold,Green} is {(Maya,Blue),(Noah,Gold),(Priya,Green),(Sam,Blue)}. Is the relation a function? Is it bijective? Write the inverse relation, then decide whether the inverse is a function.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let A={-1,0,1,2} and B={2,4,6,8}. The relation R from A to B is {(-1,a),(0,4),(1,6),(2,b)}, where a and b are elements of B and a<b. Find a and b so that R is bijective. Then write R^{-1} and explain why your values work.

WORK SPACE AND ANSWER

ANSWER KEY

Relation Switch Lab

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let R={(-3,4),(0,1),(2,4)}. Is R a function? Write R^{-1}. Is R^{-1} a function?	**R is a function. R^{-1}={(4,-3),(1,0),(4,2)}. R^{-1} is not a function because 4 maps to both -3 and 2.**
2	Let R={(1,2),(1,5),(3,4)}. Is R a function? State the ordered-pair fact that proves your answer.	**No. The input 1 maps to both 2 and 5.**
3	Let A={2,5,7}, B={a,b,c}, and R={(2,b),(5,a),(7,c)}. Is R bijective? Write R^{-1}.	**R is bijective. R^{-1}={(b,2),(a,5),(c,7)}.**
4	Let R be a relation from X={-2,0,2,4} to Y={1,3,5}: R={(-2,1),(0,3),(2,5),(4,5)}. Is R a function? Is it onto Y? Is it bijective? Is R^{-1} a function?	**R is a function and is onto Y. It is not bijective. R^{-1} is not a function because 5 maps to both 2 and 4.**
5	Let R be a relation from A={6,7,9} to B={6,9,10}: R={(6,9),(7,10),(9,6)}. Is R bijective? Write R^{-1}.	**R is bijective. R^{-1}={(9,6),(10,7),(6,9)}.**
6	The function f maps A={0,1,2,3} to B={-1,1,3,5} by f(x)=2x-1. Is f bijective? Write $f^{-1}(x)$.	**f is bijective. $f^{-1}(x)=(x+1)/2$.**

PART 2 · APPLY IT

1. **It is a function and is bijective. The inverse relation is {(P3,Inez),(P1,Kai),(P4,Lila),(P2,Omar)}.** Each student has one poster code, and every code is used exactly once. Reverse each student-code pair to form the inverse.
2. **It is a function but not bijective. The inverse relation is {(Blue,Maya),(Gold,Noah),(Green,Priya),(Blue,Sam)}. The inverse is not a function.** Every player has one room, so the relation is a function, and all three rooms are used. Blue is assigned to two players, so the relation is not one-to-one and its inverse is not a function.

CHALLENGE

a=2 and b=8. R^{-1}={(2,-1),(4,0),(6,1),(8,2)}. The outputs are 2,4,6,8, so every element of B is used once and no output repeats.

Accept inverse relations in any order. A reversed relation may be written even when it is not a function, but call it an inverse function only when each reversed input has exactly one output.