

Class Connections

Group sets using equivalence relations.

HIGH SCHOOL • MA.912.LT.5.3

MA.912.LT.5.3 "Partition a set into disjoint subsets and determine an equivalence class given the equivalence relation on a set."

NAME _____

DATE _____

BUILD THE CLASSES

An equivalence relation groups elements that share a stated rule. Each element belongs in exactly one equivalence class, so the classes are disjoint and together make the whole set.

WORKED EXAMPLE

Let $S = \{-50, -49, -48, -27, -26, -25\}$. Define $x \sim y$ when 23 divides $x - y$. Partition S into equivalence classes.

1. Numbers are related when their difference is a multiple of 23.
2. $-50 - (-27) = -23$, so $-50 \sim -27$.
3. $-49 - (-26) = -23$, and $-48 - (-25) = -23$.
4. No number in one pair differs from a number in another pair by a multiple of 23.

Answer: $\{\{-50, -27\}, \{-49, -26\}, \{-48, -25\}\}$

PRACTICE 6 problems

- 1** Let $A = \{0, 1, 2, 3, 4, 5\}$. Define $x \sim y$ when x and y have the same remainder when divided by 3. Partition A into equivalence classes.

ANSWER _____

- 2** Let $B = \{-3, -2, -1, 0, 1, 2, 3\}$. Define $x \sim y$ when x and y have the same parity. Find the equivalence class $[-2]$.

ANSWER _____

- 3** Let $C = \{6, 7, 8, 9, 10, 11\}$. Define $x \sim y$ when x and y have the same remainder when divided by 2. Find $[7]$.

ANSWER _____

- 4** Let $D = \{8, 10, 12, 14, 16, 18\}$. Define $x \sim y$ when 5 divides $x - y$. Partition D into equivalence classes.

ANSWER _____

- 5** Let $E = \{-6, -5, -4, 4, 5, 6\}$. Define $x \sim y$ when $|x| = |y|$. Find $[5]$.

ANSWER _____

- 6** A partition of F is $\{\{1, 6, 11\}, \{2, 7, 12\}, \{3, 8, 13\}, \{4, 9, 14\}, \{5, 10, 15\}\}$. Define $x \sim y$ when x and y are in the same subset. Find $[9]$.

ANSWER _____

APPLY IT Show your work

- 1** A gaming club assigns player IDs 31 through 42. Two players are in the same practice lobby when their ID numbers have the same remainder when divided by 6. List the partition of the player IDs into lobbies.

WORK SPACE

ANSWER _____

- 2** A school music festival numbers its 18 audition slots from 1 through 18. Two slots are in the same rotation group when their numbers have the same remainder when divided by 3. What is the equivalence class of slot 14?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let $G = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44\}$. Define $x \sim y$ when 7 divides $x - y$. Find every value of x for which $[x]$ has exactly 3 elements. Explain your reasoning.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Let $A = \{0, 1, 2, 3, 4, 5\}$. Define $x \sim y$ when x and y have the same remainder when divided by 3. Partition A into equivalence classes.	$\{\{0, 3\}, \{1, 4\}, \{2, 5\}\}$
2	Let $B = \{-3, -2, -1, 0, 1, 2, 3\}$. Define $x \sim y$ when x and y have the same parity. Find the equivalence class $[-2]$.	$[-2] = \{-2, 0, 2\}$
3	Let $C = \{6, 7, 8, 9, 10, 11\}$. Define $x \sim y$ when x and y have the same remainder when divided by 2. Find $[7]$.	$[7] = \{7, 9, 11\}$
4	Let $D = \{8, 10, 12, 14, 16, 18\}$. Define $x \sim y$ when 5 divides $x - y$. Partition D into equivalence classes.	$\{\{8, 18\}, \{10\}, \{12\}, \{14\}, \{16\}\}$
5	Let $E = \{-6, -5, -4, 4, 5, 6\}$. Define $x \sim y$ when $ x = y $. Find $[5]$.	$[5] = \{-5, 5\}$
6	A partition of F is $\{\{1, 6, 11\}, \{2, 7, 12\}, \{3, 8, 13\}, \{4, 9, 14\}, \{5, 10, 15\}\}$. Define $x \sim y$ when x and y are in the same subset. Find $[9]$.	$[9] = \{4, 9, 14\}$

PART 2 • APPLY IT

1. $\{\{31, 37\}, \{32, 38\}, \{33, 39\}, \{34, 40\}, \{35, 41\}, \{36, 42\}\}$ Sort the IDs by their remainders 1, 2, 3, 4, 5, and 0 when divided by 6. Each remainder group is one equivalence class.
2. $[14] = \{2, 5, 8, 11, 14, 17\}$ Slot 14 has remainder 2 when divided by 3. List every slot from 1 through 18 with remainder 2.

CHALLENGE

$x = 30, 37$, or 44 . These three numbers differ from one another by multiples of 7, so $[30] = [37] = [44] = \{30, 37, 44\}$. Every other equivalence class has 2 elements.

Accept equivalence classes and partition subsets in any order. For the challenge, accept any clear explanation that identifies the three-member class and notes that all other classes contain two elements.