LESSON MAKER PRO

Set Matchup

Use complements and matching elements to prove set equalities.

HIGH SCHOOL · MA.912.LT.5.6

MA.912.LT.5.6 "Prove set relations, including DeMorgan's Laws and equivalence relations."

NAME _____ DATE _____

MATCHING BOTH SIDES

To prove that two sets are equal, show that they contain exactly the same elements. DeMorgan's Laws say $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$, as long as complements use the same universal set.

WORKED EXAMPLE

Let $U = \{21, 22, 23, 24, 25, 26\}$, $A = \{21, 23, 25\}$, and $B = \{22, 23, 26\}$. Verify that $(A \cup B)' = A' \cap B'$.

1. $A \cup B = \{21, 22, 23, 25, 26\}$.
2. $(A \cup B)' = \{24\}$.
3. $A' = \{22, 24, 26\}$ and $B' = \{21, 24, 25\}$.
4. $A' \cap B' = \{24\}$.

Answer: $(A \cup B)' = A' \cap B' = \{24\}$.

PRACTICE 6 problems

1 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $P = \{1, 3, 4, 7\}$, and $Q = \{2, 3, 6, 7\}$. Find $(P \cap Q)'$ and $P' \cup Q'$.

ANSWER _____

2 Let $U = \{a, b, c, d, e, f\}$, $R = \{a, c, e\}$, and $S = \{b, c, d\}$. Find $(R \cup S)'$ and $R' \cap S'$.

ANSWER _____

3 Let $U = \{g, h, i, j, k, l\}$, $M = \{g, i, k\}$, and $N = \{h, i, l\}$. Are $M' \cup N'$ and $(M \cap N)'$ equivalent? Show both sets.

ANSWER _____

4 Let $U = \{14, 15, 16, 17, 18, 19\}$, $A = \{14, 16, 17\}$, and $B = \{16, 18\}$. Find $(A \setminus B)'$ and $A' \cup B$.

ANSWER _____

5 Write an equivalent membership statement for $x \notin (R \cup S)$.

ANSWER _____

6 Write a set expression for the elements that are in neither H nor J. Give an equivalent expression using complements.

ANSWER _____

APPLY IT Show your work

1 At a school hackathon, U = {Ava, Ben, Cora, Diego, Eli, Farah, Gabe, Hana}. Let C = {Ava, Cora, Diego, Gabe} be students registered for coding and R = {Ben, Cora, Farah, Gabe} be students registered for robotics. Use DeMorgan's Law to list the students registered for neither event.

WORK SPACE

ANSWER _____

2 A music app has beta testers U = {Inez, Jamal, Kai, Lena, Micah, Nora}. Let P = {Inez, Kai, Lena, Nora} be testers who saved the pop playlist, and Q = {Jamal, Kai, Micah, Nora} be testers who saved the hip-hop playlist. List the testers who are not in both groups, then state the equivalent union of complements.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Prove the set relation (A ∪ B) \ C = (A \ C) ∪ (B \ C) by using an arbitrary element x.

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ANSWER KEY

Set Matchup

HIGH SCHOOL · MA.912.LT.5.6

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let U = {1, 2, 3, 4, 5, 6, 7, 8}, P = {1, 3, 4, 7}, and Q = {2, 3, 6, 7}. Find (P ∩ Q)' and P' ∪ Q'.	**(P ∩ Q)' = {1, 2, 4, 5, 6, 8} and P' ∪ Q' = {1, 2, 4, 5, 6, 8}.**
2	Let U = {a, b, c, d, e, f}, R = {a, c, e}, and S = {b, c, d}. Find (R ∪ S)' and R' ∩ S'.	**(R ∪ S)' = {f} and R' ∩ S' = {f}.**
3	Let U = {g, h, i, j, k, l}, M = {g, i, k}, and N = {h, i, l}. Are M' ∪ N' and (M ∩ N)' equivalent? Show both sets.	**Yes. M' ∪ N' = {g, h, j, k, l} and (M ∩ N)' = {g, h, j, k, l}.**
4	Let U = {14, 15, 16, 17, 18, 19}, A = {14, 16, 17}, and B = {16, 18}. Find (A \ B)' and A' ∪ B.	**(A \ B)' = {15, 16, 18, 19} and A' ∪ B = {15, 16, 18, 19}.**
5	Write an equivalent membership statement for x ∉ (R ∪ S).	**x ∉ R and x ∉ S.**
6	Write a set expression for the elements that are in neither H nor J. Give an equivalent expression using complements.	**(H ∪ J)' = H' ∩ J'.**

PART 2 · APPLY IT

1. **{Eli, Hana}** Students in neither event are in (C ∪ R)'. Since C ∪ R = {Ava, Ben, Cora, Diego, Farah, Gabe}, its complement in U is {Eli, Hana}.
2. **{Inez, Jamal, Lena, Micah}; (P ∩ Q)' = P' ∪ Q'.** P ∩ Q = {Kai, Nora}, so its complement is {Inez, Jamal, Lena, Micah}. By DeMorgan's Law, this same set is P' ∪ Q'.

CHALLENGE

Let x be any element. x ∈ (A ∪ B) \ C means x is in A or B, and x is not in C. This means x is in A and not in C, or x is in B and not in C. Therefore x ∈ (A \ C) ∪ (B \ C), so (A ∪ B) \ C = (A \ C) ∪ (B \ C).

Accept sets written in any order, as long as every element is correct. Complements must be taken relative to the stated universal set, and equivalent verbal or symbolic membership statements are acceptable.