

Complex Plane Paths

Measure distances and locate halfway points.

HIGH SCHOOL • MA.912.NSO.2.3

MA.912.NSO.2.3 "Calculate the distance and midpoint between two numbers on the complex coordinate plane."

NAME _____

DATE _____

DISTANCE AND MIDPOINT

A complex number $a+bi$ is graphed as the point (a, b) . For two points (a, b) and (c, d) , use $d=\sqrt{(c-a)^2+(d-b)^2}$ for distance and $M=((a+c)/2, (b+d)/2)$ for midpoint.

WORKED EXAMPLE

Find the distance and midpoint between $P(-17, 13)$ and $Q(5, -9)$.

1. The horizontal change is $5-(-17)=22$, and the vertical change is $-9-13=-22$.
2. $d=\sqrt{22^2+(-22)^2}=\sqrt{968}=22\sqrt{2}$.
3. $M=(-17+5)/2, (13+(-9))/2=(-6, 2)$.

Answer: Distance: $22\sqrt{2}$; midpoint: $(-6, 2)$

PRACTICE 6 problems

- 1** For $zA=-3+6i$ and $zB=7-2i$, find the distance and midpoint.

ANSWER _____

- 2** For $zA=8i$ and $zB=6-i$, find the distance and midpoint.

ANSWER _____

- 3** For $zA=-6-7i$ and $zB=3+9i$, find the distance and midpoint.

ANSWER _____

- 4** For $zA=10$ and $zB=-8+6i$, find the distance and midpoint.

ANSWER _____

- 5** For $zA=-10+11i$ and $zB=8+3i$, find the distance and midpoint.

ANSWER _____

- 6** For $zA=9-11i$ and $zB=-3+8i$, find the distance and midpoint.

ANSWER _____

APPLY IT Show your work

- 1** A student designing an esports arena places two power-ups on a complex coordinate map. Their locations are $-13+10i$ and $11-6i$. Find the distance between the power-ups and the midpoint where a bonus item could be placed.

WORK SPACE

ANSWER _____

- 2** A robotics team maps two sensor stations on a complex coordinate plane. Station A is at $-15-8i$, and Station B is at $9+16i$. Find the distance between the stations and the midpoint for a relay station.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $zA=-16+12i$ and $zB=8-12i$, find the distance and midpoint. Then explain in one sentence why the midpoint is the same distance from both endpoints.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For $zA = -3 + 6i$ and $zB = 7 - 2i$, find the distance and midpoint.	Distance: $2\sqrt{41}$; midpoint: $(2, 2)$
2	For $zA = 8i$ and $zB = 6 - i$, find the distance and midpoint.	Distance: $3\sqrt{13}$; midpoint: $(3, 7/2)$
3	For $zA = -6 - 7i$ and $zB = 3 + 9i$, find the distance and midpoint.	Distance: $\sqrt{337}$; midpoint: $(-3/2, 1)$
4	For $zA = 10$ and $zB = -8 + 6i$, find the distance and midpoint.	Distance: $6\sqrt{10}$; midpoint: $(1, 3)$
5	For $zA = -10 + 11i$ and $zB = 8 + 3i$, find the distance and midpoint.	Distance: $2\sqrt{97}$; midpoint: $(-1, 7)$
6	For $zA = 9 - 11i$ and $zB = -3 + 8i$, find the distance and midpoint.	Distance: $\sqrt{505}$; midpoint: $(3, -3/2)$

PART 2 • APPLY IT

- Distance: $8\sqrt{13}$ units; midpoint: $(-1, 2)$** The coordinate changes are 24 and -16, so the distance is $\sqrt{24^2 + (-16)^2} = 8\sqrt{13}$. Average the coordinates to get $(-1, 2)$.
- Distance: $24\sqrt{2}$ units; midpoint: $(-3, 4)$** The coordinate changes are 24 and 24, so the distance is $\sqrt{24^2 + 24^2} = 24\sqrt{2}$. The average of the coordinates is $(-3, 4)$.

CHALLENGE

Distance: $24\sqrt{2}$; midpoint: $(-4, 0)$. The midpoint is equally far from both points because moving from it to either endpoint changes each coordinate by 12.

Accept equivalent simplified or unsimplified radical forms, such as $\sqrt{164}$ for $2\sqrt{41}$. Accept midpoint answers written as ordered pairs or as equivalent complex numbers.