LESSON MAKER PRO

Complex Plane Paths

Measure distance and locate halfway points.

HIGH SCHOOL · MA.912.NSO.2.3

MA.912.NSO.2.3 "Calculate the distance and midpoint between two numbers on the complex coordinate plane."

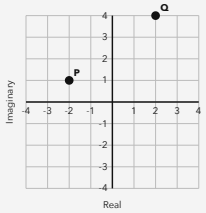
NAME _____ DATE _____

DISTANCE AND MIDPOINT

Treat a complex number a + bi as the point (a, b). Find distance with $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$ and find the midpoint by averaging the two x-coordinates and the two y-coordinates. Write the midpoint as a complex number.

WORKED EXAMPLE

Use points P and Q on the complex plane. Find their distance and midpoint.



1. Subtract coordinates: $\Delta x = 2 - (-2) = 4$ and $\Delta y = 4 - 1 = 3$.
2. Distance = $\sqrt{(4^2 + 3^2)} = \sqrt{25} = 5$.
3. Midpoint = $((-2 + 2)/2, (1 + 4)/2) = (0, 5/2)$.

Answer: Distance: 5; midpoint: 0 + 5/2i

PRACTICE 4 problems

1 Find the distance and midpoint of $z_1 = -3 + 6i$ and $z_2 = 3 - 3i$.

ANSWER _____

2 Find the distance and midpoint of $z_1 = -5 - 5i$ and $z_2 = 7 + 7i$.

ANSWER _____

3 Find the distance and midpoint of $z_1 = 6 + 5i$ and $z_2 = 6 - 3i$.

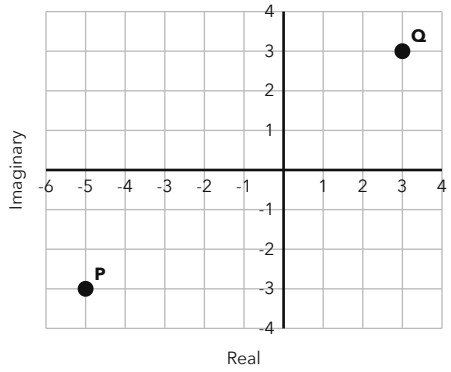
ANSWER _____

4 Find the distance and midpoint of $z_1 = -7 + 3i$ and $z_2 = 5 - 6i$.

ANSWER _____

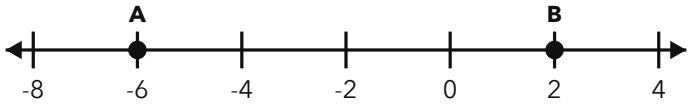
APPLY IT Show your work

1 In a music-production app, settings P and Q are plotted on the complex plane. Find the distance between them and the midpoint setting.



ANSWER _____

2 A game engine places two effects on the real axis of its complex plane. Use the marked points to find their distance and midpoint.



ANSWER _____

CHALLENGE One step up

Point P is -7 + 5i. The midpoint of P and Q is -1 - i. Find Q and the distance from P to Q.

WORK SPACE AND ANSWER

ANSWER KEY

Complex Plane Paths

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the distance and midpoint of $z_1 = -3 + 6i$ and $z_2 = 3 - 3i$.	**Distance: $3\sqrt{13}$; midpoint: $3/2i$**
2	Find the distance and midpoint of $z_1 = -5 - 5i$ and $z_2 = 7 + 7i$.	**Distance: $12\sqrt{2}$; midpoint: $1 + i$**
3	Find the distance and midpoint of $z_1 = 6 + 5i$ and $z_2 = 6 - 3i$.	**Distance: 8; midpoint: $6 + i$**
4	Find the distance and midpoint of $z_1 = -7 + 3i$ and $z_2 = 5 - 6i$.	**Distance: 15; midpoint: $-1 - 3/2i$**

PART 2 · APPLY IT

1. **Distance: 10; midpoint: $-1 + 0i$** The coordinate changes are 8 and 6, so the distance is $\sqrt{(8^2 + 6^2)} = 10$. Average the coordinates to get (-1, 0).
2. **Distance: 8; midpoint: $-2 + 0i$** Both points have imaginary coordinate 0. Their real coordinates are 8 units apart, and their average is -2.

CHALLENGE

Q = $5 - 7i$; distance: $12\sqrt{2}$

Accept midpoint answers written as ordered pairs or equivalent complex numbers, including omitted 0 parts. Accept equivalent exact radical forms only.