

Polar Power Moves

Rewrite and multiply complex numbers in trigonometric form.

HIGH SCHOOL • MA.912.NSO.2.6

MA.912.NSO.2.6 "Rewrite complex numbers to trigonometric form. Multiply complex numbers in trigonometric form."

NAME _____

DATE _____

TRIGONOMETRIC FORM RULES

Write a complex number as $r(\cos \theta + i \sin \theta)$, where r is the distance from the origin and θ shows the direction. To rewrite $a + bi$, find $r = \sqrt{a^2 + b^2}$ and use the signs of a and b to choose θ . When multiplying trigonometric forms, multiply the magnitudes and add the angles.

WORKED EXAMPLE

Write $-4 + 4i$ in trigonometric form.
Then multiply it by $7(\cos 120^\circ + i \sin 120^\circ)$.

1. $r = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$.
2. $-4 + 4i$ is in Quadrant II, so $\theta = 135^\circ$.
3. Multiply magnitudes: $(4\sqrt{2})(7) = 28\sqrt{2}$.
4. Add angles: $135^\circ + 120^\circ = 255^\circ$.

Answer: $28\sqrt{2}(\cos 255^\circ + i \sin 255^\circ)$

PRACTICE 6 problems

1 Write $3\sqrt{3} + 3i$ in trigonometric form.

ANSWER _____

2 Write $-5i$ in trigonometric form.

ANSWER _____

3 Write -9 in trigonometric form.

ANSWER _____

4 Multiply: $3(\cos 30^\circ + i \sin 30^\circ) \cdot 5(\cos 45^\circ + i \sin 45^\circ)$.

ANSWER _____

5 Multiply: $8(\cos 0^\circ + i \sin 0^\circ) \cdot 6(\cos 150^\circ + i \sin 150^\circ)$.

ANSWER _____

6 Multiply: $9(\cos 45^\circ + i \sin 45^\circ) \cdot 3(\cos 180^\circ + i \sin 180^\circ)$.

ANSWER _____

APPLY IT Show your work

- 1** In a robotics club, a control program represents two steering adjustments with the complex multipliers $6(\cos 30^\circ + i \sin 30^\circ)$ and $9(\cos 45^\circ + i \sin 45^\circ)$. What is the combined multiplier in trigonometric form?

WORK SPACE

ANSWER _____

- 2** A student DJ's sound-mixing app represents a filter by $8(\cos 150^\circ + i \sin 150^\circ)$ and an effect by $3(\cos 90^\circ + i \sin 90^\circ)$. What complex multiplier represents both settings together?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Multiply $6(\cos 30^\circ + i \sin 30^\circ)$, $3(\cos 150^\circ + i \sin 150^\circ)$, and $5(\cos 180^\circ + i \sin 180^\circ)$. Write the result using a principal angle from 0° inclusive to 360° exclusive.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write $3\sqrt{3} + 3i$ in trigonometric form.	$6(\cos 30^\circ + i \sin 30^\circ)$
2	Write $-5i$ in trigonometric form.	$5(\cos 270^\circ + i \sin 270^\circ)$
3	Write -9 in trigonometric form.	$9(\cos 180^\circ + i \sin 180^\circ)$
4	Multiply: $3(\cos 30^\circ + i \sin 30^\circ) \cdot 5(\cos 45^\circ + i \sin 45^\circ)$.	$15(\cos 75^\circ + i \sin 75^\circ)$
5	Multiply: $8(\cos 0^\circ + i \sin 0^\circ) \cdot 6(\cos 150^\circ + i \sin 150^\circ)$.	$48(\cos 150^\circ + i \sin 150^\circ)$
6	Multiply: $9(\cos 45^\circ + i \sin 45^\circ) \cdot 3(\cos 180^\circ + i \sin 180^\circ)$.	$27(\cos 225^\circ + i \sin 225^\circ)$

PART 2 • APPLY IT

1. $54(\cos 75^\circ + i \sin 75^\circ)$ Multiply the magnitudes, $6 \cdot 9 = 54$. Add the angles, $30^\circ + 45^\circ = 75^\circ$.
2. $24(\cos 240^\circ + i \sin 240^\circ)$ Multiply the magnitudes, $8 \cdot 3 = 24$. Add the angles, $150^\circ + 90^\circ = 240^\circ$.

CHALLENGE

$$90(\cos 0^\circ + i \sin 0^\circ)$$

Accept equivalent trigonometric forms whose angles differ by a multiple of 360° . For the challenge, $90(\cos 360^\circ + i \sin 360^\circ)$ is equivalent, but 0° is the requested principal angle.