LESSON MAKER PRO

Complex Number Spin

Part 1 Practice, Part 2 Apply, Part 3 Challenge

HIGH SCHOOL · MA.912.NSO.2.6

MA.912.NSO.2.6 "Rewrite complex numbers to trigonometric form. Multiply complex numbers in trigonometric form."

NAME _____ DATE _____

TRIG FORM RULE

For a nonzero complex number a + bi, find r = sqrt($a^2 + b^2$) and an angle θ for the point (a, b). Then write the number as r(cos θ + i sin θ). To multiply numbers in trigonometric form, multiply the magnitudes and add the angles.

WORKED EXAMPLE

Write z = 1 - sqrt3 i in trigonometric form. Then multiply z by q = 7(cos 100° + i sin 100°).

1. r = sqrt(1^2 + (-sqrt3)^2) = 2.
2. The point for z has argument 300°, so z = 2(cos 300° + i sin 300°).
3. Multiply magnitudes and add arguments: 2(7) = 14 and 300° + 100° = 400° = 40°.

Answer: 14(cos 40° + i sin 40°)

PRACTICE 6 problems

1 Rewrite 3 + 3i in trigonometric form, using 0° ≤ θ < 360°.

ANSWER _____

2 Rewrite -4i in trigonometric form, using 0° ≤ θ < 360°.

ANSWER _____

3 Rewrite -5 - 5sqrt3 i in trigonometric form, using 0° ≤ θ < 360°.

ANSWER _____

4 Multiply 6(cos 50° + i sin 50°) and 5(cos 80° + i sin 80°). Write the product in trigonometric form.

ANSWER _____

5 Multiply 9(cos 220° + i sin 220°) and 8(cos 70° + i sin 70°). Write the product in trigonometric form.

ANSWER _____

6 First rewrite 11 - 11i in trigonometric form. Then multiply it by 4(cos 180° + i sin 180°).

ANSWER _____

APPLY IT Show your work

1 A music production app represents two audio effects with complex gain factors. Effect A is 12(cos 75° + i sin 75°), and Effect B is 5(cos 285° + i sin 285°). The app combines the effects by multiplying the factors. What is the combined factor in trigonometric form?

WORK SPACE

ANSWER _____

2 An augmented-reality animation combines two complex scaling-and-rotation factors. The first factor is 9(cos 135° + i sin 135°), and the second is 6(cos 225° + i sin 225°). What factor does the animation use after the combination?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let z = -2 - 2i and w = 3sqrt3 + 3i. Rewrite each number in trigonometric form, then find zw in both trigonometric form and rectangular form. Explain in one sentence why the arguments are added when finding the product.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite 3 + 3i in trigonometric form, using 0° ≤ θ < 360°.	**3sqrt2(cos 45° + i sin 45°)**
2	Rewrite -4i in trigonometric form, using 0° ≤ θ < 360°.	**4(cos 270° + i sin 270°)**
3	Rewrite -5 - 5sqrt3 i in trigonometric form, using 0° ≤ θ < 360°.	**10(cos 240° + i sin 240°)**
4	Multiply 6(cos 50° + i sin 50°) and 5(cos 80° + i sin 80°). Write the product in trigonometric form.	**30(cos 130° + i sin 130°)**
5	Multiply 9(cos 220° + i sin 220°) and 8(cos 70° + i sin 70°). Write the product in trigonometric form.	**72(cos 290° + i sin 290°)**
6	First rewrite 11 - 11i in trigonometric form. Then multiply it by 4(cos 180° + i sin 180°).	**11sqrt2(cos 315° + i sin 315°); 44sqrt2(cos 135° + i sin 135°)**

PART 2 · APPLY IT

1. **60(cos 0° + i sin 0°)** Multiply the magnitudes, 12 · 5 = 60. Add the arguments, 75° + 285° = 360°, which is equivalent to 0°.
2. **54(cos 0° + i sin 0°)** Multiply the magnitudes, 9 · 6 = 54. Add the arguments, 135° + 225° = 360°, which is equivalent to 0°.

CHALLENGE

z = 2sqrt2(cos 225° + i sin 225°); w = 6(cos 30° + i sin 30°); zw = 12sqrt2(cos 255° + i sin 255°) = (6 - 6sqrt3) + (-6 - 6sqrt3)i. The arguments are added because multiplication in trigonometric form combines the rotations represented by the two factors.

Accept equivalent trigonometric forms whose magnitudes are positive and whose angles differ by a multiple of 360°. For answers requested with 0° ≤ θ < 360°, use the listed angle in that interval.