

# Vector Direction Lab

Use dot products to measure direction and find projections.

HIGH SCHOOL • MA.912.NSO.3.4

**MA.912.NSO.3.4** "Solve mathematical and real-world problems involving vectors in two dimensions using the dot product and vector projections."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## DOT, THEN PROJECT

For  $u=\langle a,b \rangle$  and  $v=\langle c,d \rangle$ , the dot product is  $u \cdot v=ac+bd$ . A dot product of 0 means the vectors are perpendicular. To project  $u$  onto  $v$ , use  $\text{proj}_v u = ((u \cdot v)/(v \cdot v))v$ .

## WORKED EXAMPLE

Find  $\text{proj}_b a$  for  $a=\langle 11,-13 \rangle$  and  $b=\langle 2,7 \rangle$ .

1.  $a \cdot b = 11(2) + (-13)(7) = -69$ .

2.  $b \cdot b = 2^2 + 7^2 = 53$ .

3.  $\text{proj}_b a = (-69/53)\langle 2,7 \rangle = \langle -138/53, -483/53 \rangle$ .

**Answer:**  $\langle -138/53, -483/53 \rangle$

## PRACTICE 6 problems

**1** Compute  $\langle 5,-3 \rangle \cdot \langle -4,9 \rangle$ .

**ANSWER** \_\_\_\_\_

**2** Compute  $\langle -6,10 \rangle \cdot \langle -5,-8 \rangle$ .

**ANSWER** \_\_\_\_\_

**3** Are  $\langle 9,15 \rangle$  and  $\langle -5,3 \rangle$  perpendicular?  
Show the dot product.

**ANSWER** \_\_\_\_\_

**4** Find  $\text{proj}_v u$ , where  $u=\langle 0,15 \rangle$  and  $v=\langle 3,4 \rangle$ .

**ANSWER** \_\_\_\_\_

**5** Find  $\text{proj}_v u$ , where  $u=\langle 20,-10 \rangle$  and  $v=\langle 5,-5 \rangle$ .

**ANSWER** \_\_\_\_\_

**6** Find  $x$  so that  $\langle x,4 \rangle$  and  $\langle 3,-6 \rangle$  are perpendicular.

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** At robotics club, a robot applies a force  $F = \langle 6, 8 \rangle$  newtons while moving through a displacement  $d = \langle 10, 5 \rangle$  meters. Use  $F \cdot d$  to find the work done.

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A video club drone has a wind vector  $w = \langle 15, 0 \rangle$  mph. Its planned flight direction is  $d = \langle 3, 4 \rangle$ . Find  $\text{proj}_d w$ , the part of the wind that pushes the drone along its flight direction.

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Let  $u = \langle 10, 15 \rangle$  and  $v = \langle 3, 4 \rangle$ . Write  $u$  as  $u_{\text{parallel}} + u_{\text{perp}}$ , where  $u_{\text{parallel}}$  is the projection of  $u$  onto  $v$  and  $u_{\text{perp}}$  is perpendicular to  $v$ .

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## PART 1 • PRACTICE

| # | PROBLEM                                                                                        | ANSWER                                         |
|---|------------------------------------------------------------------------------------------------|------------------------------------------------|
| 1 | Compute $\langle 5, -3 \rangle \cdot \langle -4, 9 \rangle$ .                                  | <b>-47</b>                                     |
| 2 | Compute $\langle -6, 10 \rangle \cdot \langle -5, -8 \rangle$ .                                | <b>-50</b>                                     |
| 3 | Are $\langle 9, 15 \rangle$ and $\langle -5, 3 \rangle$ perpendicular? Show the dot product.   | <b>Yes. The dot product is 0.</b>              |
| 4 | Find $\text{proj}_v u$ , where $u = \langle 0, 15 \rangle$ and $v = \langle 3, 4 \rangle$ .    | <b><math>\langle 36/5, 48/5 \rangle</math></b> |
| 5 | Find $\text{proj}_v u$ , where $u = \langle 20, -10 \rangle$ and $v = \langle 5, -5 \rangle$ . | <b><math>\langle 15, -15 \rangle</math></b>    |
| 6 | Find $x$ so that $\langle x, 4 \rangle$ and $\langle 3, -6 \rangle$ are perpendicular.         | <b><math>x=8</math></b>                        |

## PART 2 • APPLY IT

- 100 joules** Compute  $F \cdot d = (6)(10) + (8)(5) = 100$ . Since a newton times a meter is a joule, the work is 100 joules.
- $\langle 27/5, 36/5 \rangle$  mph** Compute  $w \cdot d = 45$  and  $d \cdot d = 25$ . Then  $\text{proj}_d w = (45/25)\langle 3, 4 \rangle = \langle 27/5, 36/5 \rangle$  mph.

## CHALLENGE

**$u_{\text{parallel}} = \langle 54/5, 72/5 \rangle$  and  $u_{\text{perp}} = \langle -4/5, 3/5 \rangle$**

Accept equivalent unsimplified fraction forms for projection coordinates. For the challenge, accept any correct decomposition in which the parallel component is a scalar multiple of  $\langle 3, 4 \rangle$  and the perpendicular component has dot product 0 with  $\langle 3, 4 \rangle$ .