LESSON MAKER PRO

Vector Direction Lab

Use dot products and projections to measure direction in two dimensions.

HIGH SCHOOL · MA.912.NSO.3.4

MA.912.NSO.3.4 "Solve mathematical and real-world problems involving vectors in two dimensions using the dot product and vector projections."

NAME _____ DATE _____

DOT PRODUCTS AND PROJECTIONS

Multiply matching coordinates and add to find a dot product: $a \cdot b = a_1b_1 + a_2b_2$. To project a onto a nonzero vector b, use $\text{proj}_b a = (a \cdot b)/(b \cdot b)b$. A dot product of 0 means the vectors are perpendicular.

WORKED EXAMPLE

Find $\text{proj}_b a$ for a = <12, -9> and b = <8, 7>.

1. Find the dot product: $a \cdot b = 12(8) + (-9)(7) = 33$.
2. Find $b \cdot b$: $8(8) + 7(7) = 113$.
3. Substitute: $\text{proj}_b a = (33/113)\langle 8, 7\rangle = \langle 264/113, 231/113\rangle$.
4.

Answer: $\text{proj}_b a = \langle 264/113, 231/113\rangle$

PRACTICE 6 problems

1 Find the dot product of <4, -5> and <-1, 6>.

ANSWER _____

2 Find the dot product of <1, 6> and <10, -4>.

ANSWER _____

3 Find $\text{proj}_v u$ when u = <2, 1> and v = <4, -6>.

ANSWER _____

4 Are a = <5, 2> and b = <1, -4> perpendicular? Show the dot product that supports your answer.

ANSWER _____

_____

5 Find the scalar projection of a = <10, -2> onto b = <1, 4>.

ANSWER _____

6 Find the value of k that makes <k, 6> perpendicular to <5, -1>.

ANSWER _____

APPLY IT Show your work

1 In a video game, a character's intended movement is u = <14, 2> tiles. The safe-path direction is v = <1, 2>. Find the vector projection of u onto v. Then find the remaining movement vector, u - $\text{proj}_v u$.

WORK SPACE

ANSWER _____

2 A robotics club's robot moves by the displacement <10, -6> meters. Its planned route points in the direction <4, -1>. How far does the robot move in the planned route direction? State whether that motion is forward or backward along the route.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Vector a = <x, 4> is projected onto b = <2, 1>. The projection has length $3\sqrt{5}$ and points in the same direction as b. Find x and explain why the direction condition matters.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the dot product of <4, -5> and <-1, 6>.	**-34**
2	Find the dot product of <1, 6> and <10, -4>.	**-14**
3	Find $\text{proj}_v u$ when u = <2, 1> and v = <4, -6>.	**<2/13, -3/13>**
4	Are a = <5, 2> and b = <1, -4> perpendicular? Show the dot product that supports your answer.	**No. a · b = -3.**
5	Find the scalar projection of a = <10, -2> onto b = <1, 4>.	**2/√17**
6	Find the value of k that makes <k, 6> perpendicular to <5, -1>.	**k = 6/5**

PART 2 · APPLY IT

1. **$\text{proj}_v u$ = <18/5, 36/5>; u - $\text{proj}_v u$ = <52/5, -26/5>** Compute u · v = 18 and v · v = 5, then multiply <1, 2> by 18/5. Subtract the projection vector from <14, 2>.
2. **46/√17 meters, forward along the route** The scalar projection is [<10, -6> · <4, -1>]/√17 = 46/√17. The dot product is positive, so the motion is forward.

CHALLENGE

x = 11/2. Since the projection points in the same direction as b, its scalar projection is positive, so (2x + 4)/√5 = 3√5. This gives 2x + 4 = 15, so x = 11/2.

Accept equivalent exact forms, including rationalized forms such as 2√17/17 and 46√17/17. For vector projections, accept coordinate fractions that are algebraically equivalent.