

Vector Power Moves

Use dot products and cross products in three dimensions.

HIGH SCHOOL · MA.912.NSO.3.5

MA.912.NSO.3.5 "Solve mathematical and real-world problems involving vectors in three dimensions using the dot product and cross product."

NAME _____

DATE _____

TWO VECTOR TOOLS

For $u = \langle a, b, c \rangle$ and $v = \langle d, e, f \rangle$, the dot product is $u \cdot v = ad + be + cf$. The cross product is $u \times v = \langle bf - ce, cd - af, ae - bd \rangle$; it is perpendicular to both vectors, and its magnitude gives the area of the parallelogram formed by the vectors.

WORKED EXAMPLE

Let $u = \langle 2, -3, 5 \rangle$ and $v = \langle 4, 1, -2 \rangle$.
Find $u \cdot v$ and $u \times v$.

1. $u \cdot v = 2(4) + (-3)(1) + 5(-2) = -5$.

2. $u \times v = \langle (-3)(-2) - 5(1), 5(4) - 2(-2), 2(1) - (-3)(4) \rangle$.

3. $u \times v = \langle 1, 24, 14 \rangle$.

Answer: $u \cdot v = -5$; $u \times v = \langle 1, 24, 14 \rangle$.

PRACTICE 6 problems

- 1** Find the dot product of $u = \langle 6, -7, 8 \rangle$ and $v = \langle 9, 10, -11 \rangle$.

ANSWER _____

- 2** Find $a \times b$ for $a = \langle 6, 7, 8 \rangle$ and $b = \langle 9, -10, 11 \rangle$.

ANSWER _____

- 3** Decide whether $p = \langle 6, 8, 10 \rangle$ and $q = \langle 8, -6, 0 \rangle$ are perpendicular. Show the dot product.

ANSWER _____

- 4** Find the area of the parallelogram formed by $a = \langle 7, 0, 0 \rangle$ and $b = \langle 0, 8, 0 \rangle$.

ANSWER _____

- 5** Use the dot product to classify the angle between $m = \langle 9, 10, 0 \rangle$ and $n = \langle -11, 8, 0 \rangle$ as acute, obtuse, or right.

ANSWER _____

- 6** Find a normal vector to the plane determined by $r = \langle 6, 0, 8 \rangle$ and $s = \langle 0, 10, 0 \rangle$. Use $r \times s$.

ANSWER _____

APPLY IT Show your work

- 1** A virtual-reality game models a controller's force as $F = \langle 7, -8, 9 \rangle$ newtons and its displacement as $d = \langle 10, 11, 12 \rangle$ meters. Find the work done by the force.

WORK SPACE

ANSWER _____

- 2** The theater club designs a parallelogram-shaped light frame. Two edge vectors from one corner are $a = \langle 9, 0, 12 \rangle$ centimeters and $b = \langle 0, 15, 0 \rangle$ centimeters. Find the area of the frame.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Vectors $a = \langle 6, x, 8 \rangle$ and $b = \langle 10, -6, 0 \rangle$ are perpendicular. Find x . Then find $a \times b$ and the area of the parallelogram formed by a and b . Show your reasoning.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the dot product of $u=\langle 6,-7,8 \rangle$ and $v=\langle 9,10,-11 \rangle$.	-104
2	Find $a \times b$ for $a=\langle 6,7,8 \rangle$ and $b=\langle 9,-10,11 \rangle$.	$\langle 157,6,-123 \rangle$
3	Decide whether $p=\langle 6,8,10 \rangle$ and $q=\langle 8,-6,0 \rangle$ are perpendicular. Show the dot product.	Yes; $p \cdot q = 0$.
4	Find the area of the parallelogram formed by $a=\langle 7,0,0 \rangle$ and $b=\langle 0,8,0 \rangle$.	56 square units
5	Use the dot product to classify the angle between $m=\langle 9,10,0 \rangle$ and $n=\langle -11,8,0 \rangle$ as acute, obtuse, or right.	Obtuse, because $m \cdot n = -19$.
6	Find a normal vector to the plane determined by $r=\langle 6,0,8 \rangle$ and $s=\langle 0,10,0 \rangle$. Use $r \times s$.	$\langle -80,0,60 \rangle$

PART 2 · APPLY IT

- 90 joules** Use the dot product: $F \cdot d = 7(10) + (-8)(11) + 9(12) = 90$.
- 225 square centimeters** Find $a \times b = \langle -180,0,135 \rangle$. Its magnitude is 225, which is the parallelogram's area.

CHALLENGE

$x=10$; $a \times b = \langle 48,80,-136 \rangle$; area= $40\sqrt{17}$ square units

For perpendicular and angle-classification items, require dot-product evidence. Accept equivalent arithmetic work, but require the requested cross-product direction when a specific order is given.