

Matrix Mission

Represent systems and solve them with row operations.

HIGH SCHOOL • MA.912.NSO.4.2

MA.912.NSO.4.2 "Given a mathematical or real-world context, represent and solve a system of two- or three-variable linear equations using matrices."

NAME _____

DATE _____

ROW REDUCTION

Write the coefficients and constants in an augmented matrix, keeping each variable column in the same order. Use row swaps, multiplying a row by a nonzero number, and replacing a row with a sum or difference of rows to simplify the matrix. For a unique solution, the coefficient columns can reduce to an identity matrix, while a zero row or a contradictory row must be interpreted from the context.

WORKED EXAMPLE

Solve $2x + 3y = 7$ and $x - y = 1$ with an augmented matrix.

1. Start: $[2 \ 3 \ | \ 7; \ 1 \ -1 \ | \ 1]$
2. $R1 \leftarrow R1 - 2R2$: $[0 \ 5 \ | \ 5; \ 1 \ -1 \ | \ 1]$
3. $R1 \leftarrow (1/5)R1$: $[0 \ 1 \ | \ 1; \ 1 \ -1 \ | \ 1]$
4. $R2 \leftarrow R2 + R1$: $[0 \ 1 \ | \ 1; \ 1 \ 0 \ | \ 2]$

Answer: $x = 2, y = 1$

PRACTICE 6 problems

- 1** Use an augmented matrix to solve: $4x + 8y = 64$ and $8x - 4y = 8$.

ANSWER _____

- 2** Use an augmented matrix to solve: $6x + 4y = 60$ and $4x - 8y = -24$.

ANSWER _____

- 3** Use an augmented matrix to solve: $4x + 6y + 8z = 116$, $8x - 4y + 6z = 56$, and $6x + 8y - 4z = 40$.

ANSWER _____

- 4** Use an augmented matrix to determine whether this system has one solution, no solution, or infinitely many solutions: $4x + 8y = 24$ and $6x + 12y = 42$.

ANSWER _____

- 5** Use an augmented matrix to solve and describe all solutions: $4x + 16y = 48$ and $8x + 32y = 96$.

ANSWER _____

- 6** Solve the augmented matrix: $[6 \ 4 \ | \ 52; \ 4 \ -6 \ | \ 0]$.

ANSWER _____

APPLY IT Show your work

- 1** At a gaming tournament, standard passes cost \$8 and VIP passes cost \$16. A group buys 28 passes for \$336. Let x be the number of standard passes and y be the number of VIP passes. Represent the situation with an augmented matrix and solve.

WORK SPACE

ANSWER _____

- 2** A school club compares three print-shop invoices. A hoodie costs x dollars, a T-shirt costs y dollars, and a sticker costs z dollars. An invoice for 4 hoodies, 8 T-shirts, and 6 stickers is \$108. An invoice for 6 hoodies, 4 T-shirts, and 8 stickers is \$110. An invoice for 8 hoodies, 6 T-shirts, and 4 stickers is \$124. Represent the invoices with an augmented matrix and solve.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Use an augmented matrix to determine whether this system is consistent. Explain what the final row tells you: $4x + 8y + 12z = 48$, $6x + 12y + 18z = 72$, and $8x + 16y + 24z = 104$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Use an augmented matrix to solve: $4x + 8y = 64$ and $8x - 4y = 8$.	$x = 4, y = 6$
2	Use an augmented matrix to solve: $6x + 4y = 60$ and $4x - 8y = -24$.	$x = 6, y = 6$
3	Use an augmented matrix to solve: $4x + 6y + 8z = 116$, $8x - 4y + 6z = 56$, and $6x + 8y - 4z = 40$.	$x = 4, y = 6, z = 8$
4	Use an augmented matrix to determine whether this system has one solution, no solution, or infinitely many solutions: $4x + 8y = 24$ and $6x + 12y = 42$.	No solution
5	Use an augmented matrix to solve and describe all solutions: $4x + 16y = 48$ and $8x + 32y = 96$.	Infinitely many solutions: $y = t, x = 12 - 4t$
6	Solve the augmented matrix: $[6 \ 4 \mid 52; 4 \ -6 \mid 0]$.	$x = 6, y = 4$

PART 2 • APPLY IT

- 14 standard passes and 14 VIP passes** Use $x + y = 28$ and $8x + 16y = 336$. Row reduce the augmented matrix to find $x = 14$ and $y = 14$.
- Hoodie: \$9, T-shirt: \$6, sticker: \$4** Write one equation for each invoice and row reduce the three-variable augmented matrix. The reduced matrix gives $x = 9, y = 6$, and $z = 4$.

CHALLENGE

No solution. Row reduction produces a row equivalent to $[0 \ 0 \ 0 \mid 8]$, which means $0 = 8$, a contradiction.

Accept any correct row-reduction path and equivalent descriptions of the solution set in the dependent system. For the challenge, accept an explanation that identifies the contradictory zero-coefficient row with a nonzero constant.