LESSON MAKER PRO

Matrix Mission

Represent systems and solve them with row operations.

HIGH SCHOOL · MA.912.NSO.4.2

MA.912.NSO.4.2 "Given a mathematical or real-world context, represent and solve a system of two- or three-variable linear equations using matrices."

NAME _____ DATE _____

ROW REDUCTION

Write the coefficients and constants in an augmented matrix, keeping each variable column in the same order. Use row swaps, multiplying a row by a nonzero number, and replacing a row with a sum or difference of rows to simplify the matrix. For a unique solution, the coefficient columns can reduce to an identity matrix, while a zero row or a contradictory row must be interpreted from the context.

WORKED EXAMPLE

Solve $2x + 3y = 7$ and $x - y = 1$ with an augmented matrix.

1. Start: [2 3 | 7; 1 -1 | 1]
2. R1 <- R1 - 2R2: [0 5 | 5; 1 -1 | 1]
3. R1 <- (1/5)R1: [0 1 | 1; 1 -1 | 1]
4. R2 <- R2 + R1: [0 1 | 1; 1 0 | 2]

Answer: x = 2, y = 1

PRACTICE 6 problems

1 Use an augmented matrix to solve: 4x + 8y = 64 and 8x - 4y = 8.

ANSWER _____

2 Use an augmented matrix to solve: 6x + 4y = 60 and 4x - 8y = -24.

ANSWER _____

3 Use an augmented matrix to solve: 4x + 6y + 8z = 116, 8x - 4y + 6z = 56, and 6x + 8y - 4z = 40.

ANSWER _____

4 Use an augmented matrix to determine whether this system has one solution, no solution, or infinitely many solutions: 4x + 8y = 24 and 6x + 12y = 42.

ANSWER _____

5 Use an augmented matrix to solve and describe all solutions: 4x + 16y = 48 and 8x + 32y = 96.

ANSWER _____

6 Solve the augmented matrix: [6 4 | 52; 4 -6 | 0].

ANSWER _____

APPLY IT Show your work

1 At a gaming tournament, standard passes cost $8 and VIP passes cost $16. A group buys 28 passes for $336. Let x be the number of standard passes and y be the number of VIP passes. Represent the situation with an augmented matrix and solve.

WORK SPACE

ANSWER _____

2 A school club compares three print-shop invoices. A hoodie costs x dollars, a T-shirt costs y dollars, and a sticker costs z dollars. An invoice for 4 hoodies, 8 T-shirts, and 6 stickers is $108. An invoice for 6 hoodies, 4 T-shirts, and 8 stickers is $110. An invoice for 8 hoodies, 6 T-shirts, and 4 stickers is $124. Represent the invoices with an augmented matrix and solve.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Use an augmented matrix to determine whether this system is consistent. Explain what the final row tells you: $4x + 8y + 12z = 48$, $6x + 12y + 18z = 72$, and $8x + 16y + 24z = 104$.

WORK SPACE AND ANSWER

ANSWER KEY

Matrix Mission

HIGH SCHOOL · MA.912.NSO.4.2

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use an augmented matrix to solve: 4x + 8y = 64 and 8x - 4y = 8.	**x = 4, y = 6**
2	Use an augmented matrix to solve: 6x + 4y = 60 and 4x - 8y = -24.	**x = 6, y = 6**
3	Use an augmented matrix to solve: 4x + 6y + 8z = 116, 8x - 4y + 6z = 56, and 6x + 8y - 4z = 40.	**x = 4, y = 6, z = 8**
4	Use an augmented matrix to determine whether this system has one solution, no solution, or infinitely many solutions: 4x + 8y = 24 and 6x + 12y = 42.	**No solution**
5	Use an augmented matrix to solve and describe all solutions: 4x + 16y = 48 and 8x + 32y = 96.	**Infinitely many solutions: y = t, x = 12 - 4t**
6	Solve the augmented matrix: [6 4 | 52; 4 -6 | 0].	**x = 6, y = 4**

PART 2 · APPLY IT

1. **14 standard passes and 14 VIP passes** Use x + y = 28 and 8x + 16y = 336. Row reduce the augmented matrix to find x = 14 and y = 14.
2. **Hoodie: $9, T-shirt: $6, sticker: $4** Write one equation for each invoice and row reduce the three-variable augmented matrix. The reduced matrix gives x = 9, y = 6, and z = 4.

CHALLENGE

No solution. Row reduction produces a row equivalent to [0 0 0 | 8], which means 0 = 8, a contradiction.

Accept any correct row-reduction path and equivalent descriptions of the solution set in the dependent system. For the challenge, accept an explanation that identifies the contradictory zero-coefficient row with a nonzero constant.