

Matrix Power Moves

Use determinants and inverses to solve matrix problems.

HIGH SCHOOL • MA.912.NSO.4.4

MA.912.NSO.4.4 "Solve mathematical and real-world problems using the inverse and determinant of matrices."

NAME _____

DATE _____

INVERT IT

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det(A) = ad - bc$. If $\det(A)$ is not 0, then $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. To solve $A\begin{bmatrix} x \\ y \end{bmatrix}^T = b$, multiply both sides by A^{-1} .

WORKED EXAMPLE

Find A^{-1} for $A = \begin{bmatrix} 11 & 13 \\ 17 & 19 \end{bmatrix}$.

- $\det(A) = (11)(19) - (13)(17) = 209 - 221 = -12$.
- Swap 11 and 19, then change the signs of 13 and 17: $\begin{bmatrix} 19 & -13 \\ -17 & 11 \end{bmatrix}$.
- Multiply each entry by $-1/12$.

Answer: $A^{-1} = \begin{bmatrix} -19/12 & 13/12 \\ 17/12 & -11/12 \end{bmatrix}$

PRACTICE 6 problems

- 1** Find $\det(A)$ for $A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$.

ANSWER _____

- 2** Find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

ANSWER _____

- 3** Use an inverse matrix to solve the system:
 $2x + y = 7$ and $x + y = 5$.

ANSWER _____

- 4** Is $A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ invertible? Show your determinant.

ANSWER _____

- 5** Find the inverse of $A = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$.

ANSWER _____

- 6** Use an inverse matrix to solve the system: $x - y = 1$ and $4x + 2y = 14$.

ANSWER _____

APPLY IT Show your work

- 1** At a school concert, student tickets cost \$6 and adult tickets cost \$10. The box office sold 16 tickets for \$128. Use an inverse matrix to find how many student tickets and adult tickets were sold.

WORK SPACE

ANSWER _____

- 2** A gaming club buys hoodies for \$14 each and T-shirts for \$8 each. The club buys 15 items for a total of \$162. Use an inverse matrix to find how many hoodies and T-shirts the club buys.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For which values of k does the system with coefficient matrix $A = \begin{bmatrix} k & 2 \\ 3 & 6 \end{bmatrix}$ have a unique solution? Explain using the determinant.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\det(A)$ for $A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$.	3
2	Find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.	$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$
3	Use an inverse matrix to solve the system: $2x + y = 7$ and $x + y = 5$.	$x = 2, y = 3$
4	Is $A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ invertible? Show your determinant.	No. $\det(A) = 0$, so A is not invertible.
5	Find the inverse of $A = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$.	$A^{-1} = \begin{bmatrix} -1/6 & 1/3 \\ 1/2 & 0 \end{bmatrix}$
6	Use an inverse matrix to solve the system: $x - y = 1$ and $4x + 2y = 14$.	$x = 8/3, y = 5/3$

PART 2 · APPLY IT

- 8 student tickets and 8 adult tickets** Write the system $6s + 10a = 128$ and $s + a = 16$. Invert the coefficient matrix $\begin{bmatrix} 6 & 10 \\ 1 & 1 \end{bmatrix}$ to get $s = 8$ and $a = 8$.
- 7 hoodies and 8 T-shirts** Write the system $14h + 8t = 162$ and $h + t = 15$. Invert the coefficient matrix $\begin{bmatrix} 14 & 8 \\ 1 & 1 \end{bmatrix}$ to get $h = 7$ and $t = 8$.

CHALLENGE

The system has a unique solution when $k \neq 1$. $\det(A) = 6k - 6 = 6(k - 1)$, which is nonzero only when $k \neq 1$.

Accept equivalent simplified inverse entries or an inverse written as a scalar times an adjugate matrix. For system problems, accept the ordered pair form for x and y .