

Identity Power Up

Use Pythagorean identities to find and simplify trig ratios.

HIGH SCHOOL • MA.912.T.1.5

MA.912.T.1.5 "Prove Pythagorean Identities. Apply Pythagorean Identities to calculate trigonometric ratios and to solve problems."

NAME _____

DATE _____

THE BIG IDEA

The identity $\sin^2 \theta + \cos^2 \theta = 1$ connects sine and cosine. Dividing this identity by $\cos^2 \theta$ gives $1 + \tan^2 \theta = \sec^2 \theta$, and dividing by $\sin^2 \theta$ gives $1 + \cot^2 \theta = \csc^2 \theta$. Use the quadrant or a sign clue when you take a square root.

WORKED EXAMPLE

Given $\cos \theta = 8/17$ and θ is in Quadrant I, find $\sin \theta$.

1. $\sin^2 \theta + \cos^2 \theta = 1$
2. $\sin^2 \theta = 1 - (8/17)^2$
3. $\sin^2 \theta = 225/289$
4. $\sin \theta = 15/17$ because sine is positive in Quadrant I.

Answer: $\sin \theta = 15/17$

PRACTICE 6 problems

- 1** Given $\sin \theta = 3/5$ and θ is in Quadrant I, find $\cos \theta$.

ANSWER _____

- 2** Given $\cos \theta = -12/13$ and θ is in Quadrant II, find $\sin \theta$.

ANSWER _____

- 3** Given $\tan \theta = 7/24$ and θ is in Quadrant I, find $\sec \theta$.

ANSWER _____

- 4** Given $\cot \theta = -9/40$ and $\sin \theta < 0$, find $\cos \theta$.

ANSWER _____

- 5** Simplify: $(1 - \sin^2 \theta)/\cos^2 \theta$

ANSWER _____

- 6** Simplify: $\csc^2 \theta - \cot^2 \theta$

ANSWER _____

APPLY IT Show your work

- 1** At a skate park, a ramp makes an angle θ with the ground. The ramp design has $\sin \theta = \frac{9}{41}$. Find $\cos \theta$, which compares the horizontal run to the ramp length.

WORK SPACE

ANSWER _____

- 2** In a racing video game, a car climbs a virtual hill at an angle θ . The game model uses $\tan \theta = \frac{3}{4}$. Find $\sec \theta$, the factor that compares the hill length to the horizontal distance.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Prove that $(1 - \cos^2 \theta) / \sin \theta = \sin \theta$ for $\sin \theta \neq 0$. Write each algebra step.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given $\sin \theta = 3/5$ and θ is in Quadrant I, find $\cos \theta$.	$\cos \theta = 4/5$
2	Given $\cos \theta = -12/13$ and θ is in Quadrant II, find $\sin \theta$.	$\sin \theta = 5/13$
3	Given $\tan \theta = 7/24$ and θ is in Quadrant I, find $\sec \theta$.	$\sec \theta = 25/24$
4	Given $\cot \theta = -9/40$ and $\sin \theta < 0$, find $\cos \theta$.	$\cos \theta = 9/41$
5	Simplify: $(1 - \sin^2 \theta)/\cos^2 \theta$	1
6	Simplify: $\csc^2 \theta - \cot^2 \theta$	1

PART 2 · APPLY IT

- $\cos \theta = 40/41$** Use $\sin^2 \theta + \cos^2 \theta = 1$. Since the ramp angle is acute, take the positive square root.
- $\sec \theta = 5/4$** Use $1 + \tan^2 \theta = \sec^2 \theta$. The hill angle is acute, so $\sec \theta$ is positive.

CHALLENGE

$1 - \cos^2 \theta = \sin^2 \theta$, so $(1 - \cos^2 \theta)/\sin \theta = \sin^2 \theta/\sin \theta = \sin \theta$.

Accept equivalent forms, such as positive square-root forms when the stated quadrant or context requires a positive ratio. For the challenge, accept any valid substitution using $\sin^2 \theta + \cos^2 \theta = 1$ followed by correct cancellation with $\sin \theta \neq 0$.