LESSON MAKER PRO

Identity Power Moves

Use Pythagorean identities to find and simplify trigonometric ratios.

HIGH SCHOOL · MA.912.T.1.5

MA.912.T.1.5 "Prove Pythagorean Identities. Apply Pythagorean Identities to calculate trigonometric ratios and to solve problems."

NAME _____ DATE _____

THE CORE CONNECTION

For every angle x, $\sin^2 x + \cos^2 x = 1$. Substitute a known ratio into this identity to find a missing ratio, then use the angle's quadrant to choose the correct sign. Dividing the identity by $\sin^2 x$ gives $1 + \cot^2 x = \csc^2 x$.

WORKED EXAMPLE

Given $\cos x = 5/13$ and x is acute, find $\sin x$.

1. Start with $\sin^2 x + \cos^2 x = 1$.
2. Substitute: $\sin^2 x = 1 - (5/13)^2 = 144/169$.
3. Because x is acute, $\sin x$ is positive, so $\sin x = 12/13$.

Answer: $\sin x = 12/13$

PRACTICE 6 problems

1 Given $\sin x = 8/17$ and x is acute, find $\cos x$.

ANSWER _____

2 Given $\cos x = -7/25$ and x is in Quadrant II, find $\sin x$.

ANSWER _____

3 Given $\tan x = 8/15$ and x is acute, find $\sec x$.

ANSWER _____

4 Prove $1 + \cot^2 x = \csc^2 x$ by starting with $\sin^2 x + \cos^2 x = 1$. State what you divide by.

ANSWER _____

_____

5 Given $\sec x = -25/7$ and x is in Quadrant III, find $\tan x$.

ANSWER _____

6 Simplify $(1 - \cos^2 x)\sec^2 x$.

ANSWER _____

APPLY IT Show your work

1 A video game designer models a ski-jump path with an acute launch angle x. The vertical part of each 29-unit move is 20 units, so sin x = 20/29. What is cos x, the horizontal part per unit of path?

WORK SPACE

ANSWER _____

2 A student helping at a school concert designs a ramp that rises 7 feet for every 24 feet of horizontal run. Let x be the acute angle between the ramp and the ground. Find sec x and the ramp length when the horizontal run is 24 feet.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For an angle x in Quadrant II, sin x + cos x = 1/3. Use $\sin^2 x + \cos^2 x = 1$ to determine tan x. Show how the identity gives the value of sin x cos x.

WORK SPACE AND ANSWER

ANSWER KEY

Identity Power Moves

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given sin x = 8/17 and x is acute, find cos x.	**cos x = 15/17**
2	Given cos x = -7/25 and x is in Quadrant II, find sin x.	**sin x = 24/25**
3	Given tan x = 8/15 and x is acute, find sec x.	**sec x = 17/15**
4	Prove $1 + \cot^2 x = \csc^2 x$ by starting with $\sin^2 x + \cos^2 x = 1$. State what you divide by.	**Divide by $\sin^2 x$: $1 + \cos^2 x/\sin^2 x = 1/\sin^2 x$, so $1 + \cot^2 x = \csc^2 x$.**
5	Given sec x = -25/7 and x is in Quadrant III, find tan x.	**tan x = 24/7**
6	Simplify $(1 - \cos^2 x)\sec^2 x$.	**$\tan^2 x$**

PART 2 · APPLY IT

1. **cos x = 21/29** Use $\cos^2 x = 1 - \sin^2 x$. Since x is acute, choose the positive value, 21/29.
2. **sec x = 25/24; ramp length = 25 feet** tan x = 7/24, so $1 + \tan^2 x = \sec^2 x$ gives sec x = 25/24. Since sec x is ramp length divided by horizontal run, the ramp length is 25 feet.

CHALLENGE

sin x cos x = -4/9; tan x = -(9 + sqrt(17))/8

Accept algebraically equivalent forms of the challenge answer, including (1 + sqrt(17))/(1 - sqrt(17)). For identity proofs, require the stated division and correct conversion of ratios to cotangent and cosecant.