Angle Formula Lab

Use identities to find exact values and justify each step.

HIGH SCHOOL · MA.912.T.1.6

MA.912.T.1.6 "Prove the Double-Angle, Half-Angle, Angle Sum and Difference formulas for sine, cosine, and tangent. Apply these formulas to solve problems."

NAME ____________________ DATE ____________

BUILD AND REVERSE IDENTITIES

For sums and differences, sin(a±b)=sin a cos b±cos a sin b, cos(a±b)=cos a cos b∓sin a sin b, and tan(a±b)=(tan a±tan b)/(1∓tan a tan b). Setting b=a gives sin(2a)=2 sin a cos a, cos(2a)=cos^2 a-sin^2 a, and tan(2a)=2 tan a/(1-tan^2 a). For half-angles, sin(a/2)=±sqrt((1-cos a)/2), cos(a/2)=±sqrt((1+cos a)/2), and tan(a/2)=sin a/(1+cos a), where defined, so choose the sign from the quadrant.

WORKED EXAMPLE

If tan p=9/14 and tan q=4/13, find tan(p+q).

1. tan(p+q)=(tan p+tan q)/(1-tan p tan q).
2. tan(p+q)=(9/14+4/13)/(1-(9/14)(4/13)).
3. tan(p+q)=(173/182)/(146/182).
4. tan(p+q)=173/146.

Answer: 173/146

PRACTICE

6 problems

1 Use the cosine sum formula to prove that cos(2a)=cos^2 a-sin^2 a.

ANSWER ____________________

2 If sin x=5/12 and x is in Quadrant I, find cos(2x).

ANSWER ____________________

3 If cos y=-3/5 and y is in Quadrant II, find sin(y/2).

ANSWER ____________________

4 If tan r=3/8 and tan s=1/6, find tan(r-s).

ANSWER ____________________

5 If cos z=12/37 and z is in Quadrant IV, find sin(z/2).

ANSWER ____________________

6 For cos m not equal to 0, simplify sin(2m)/(1+cos(2m)).

ANSWER ____________________

APPLY IT
Show your work

1 For a video project, a student combines two upward camera tilt angles, a and b. Both angles are acute, tan a=7/12, and tan b=2/5. What is tan(a+b)?

WORK SPACE

ANSWER ______________________

2 At a robotics competition, a team programs an arm to rotate through an angle theta, where 90 degrees<theta<180 degrees and cos theta=-15/17. The next command rotates the arm through theta/2. What is cos(theta/2)?

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

If sin x=8/17 and x is in Quadrant II, find tan(x/2) and sin(2x). Explain how the quadrant determines the signs you use.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the cosine sum formula to prove that cos(2a)=cos^2 a-sin^2 a.	**cos(a+a)=cos a cos a-sin a sin a=cos^2 a-sin^2 a. Therefore, cos(2a)=cos^2 a-sin^2 a.**
2	If sin x=5/12 and x is in Quadrant I, find cos(2x).	**47/72**
3	If cos y=-3/5 and y is in Quadrant II, find sin(y/2).	**2sqrt5/5**
4	If tan r=3/8 and tan s=1/6, find tan(r-s).	**10/51**
5	If cos z=12/37 and z is in Quadrant IV, find sin(z/2).	**5sqrt74/74**
6	For cos m not equal to 0, simplify sin(2m)/(1+cos(2m)).	**tan m**

PART 2 · APPLY IT

1. **59/46** Use tan(a+b)=(tan a+tan b)/(1-tan a tan b). Substitution gives (7/12+2/5)/(1-(7/12)(2/5))=(59/60)/(46/60)=59/46.
2. **sqrt17/17** Because theta/2 is in Quadrant I, use the positive half-angle value. cos(theta/2)=sqrt((1+cos theta)/2)=sqrt((1-15/17)/2)=sqrt17/17.

CHALLENGE

tan(x/2)=4 and sin(2x)=-240/289. Since x is in Quadrant II, cos x=-15/17. Also, x/2 is in Quadrant I, so tan(x/2) is positive.

Accept equivalent exact radical forms, such as 5/sqrt74 for problem 5 if rationalization is not required. For proof items, require a valid substitution into the stated sum, difference, or double-angle identity and correct quadrant signs.