LESSON MAKER PRO

Exact Angle Toolkit

Use angle formulas to find exact values and justify identities.

HIGH SCHOOL · MA.912.T.1.6

MA.912.T.1.6 "Prove the Double-Angle, Half-Angle, Angle Sum and Difference formulas for sine, cosine, and tangent. Apply these formulas to solve problems."

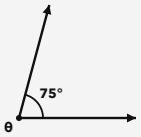
NAME _____ DATE _____

FORMULA CONNECTIONS

For angles α and β, use sin(α±β)=sin α cos β±cos α sin β, cos(α±β)=cos α cos β∓sin α sin β, and tan(α±β)=(tan α±tan β)/(1∓tan α tan β). A double angle uses the same angle twice: sin 2α=2 sin α cos α, cos 2α=cos² α-sin² α, and tan 2α=2 tan α/(1-tan² α). For half angles, sin(α/2)=±sqrt((1-cos α)/2), cos(α/2)=±sqrt((1+cos α)/2), and tan(α/2)=sin α/(1+cos α); choose the sign from the quadrant of α/2.

WORKED EXAMPLE

Find the exact value of sin θ for the angle shown. Rewrite θ as 45°+30°.

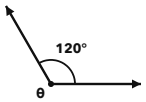


1. sin(45°+30°)=sin 45° cos 30°+cos 45° sin 30°.
2. = (sqrt2/2)(sqrt3/2)+(sqrt2/2)(1/2).
3. = (sqrt6+sqrt2)/4.

Answer: (sqrt6+sqrt2)/4

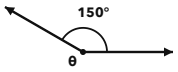
PRACTICE 6 problems

1 The angle shown is θ. Use a double-angle formula to find the exact value of cos θ.



ANSWER _____

2 Use an angle-sum formula to find the exact value of sin θ for the angle shown.



ANSWER _____

3 Use the tangent double-angle formula to find the exact value of tan 120°. Start with tan 60°=sqrt3.

ANSWER _____

4 Given cos 105°=(sqrt2-sqrt6)/4, find the exact value of cos(105°/2). Note that 105°/2 is in Quadrant I.

ANSWER _____

5 Use the sine angle-sum formula to simplify sin(x+x). Show the substitution and the simplified result.

ANSWER _____

6 Solve for x, where 0°≤x≤90°: sin(x+90°)=1/2.

ANSWER _____

APPLY IT Show your work

1 A game designer makes a character move 10 units at an angle of 120° counterclockwise from the positive horizontal direction. What is the character's exact horizontal change in position? Use a double-angle formula.

WORK SPACE

ANSWER _____

2 A concert spotlight starts aimed horizontally and sweeps upward through 120°. At the halfway position, its 12 m beam points at 60° above horizontal. Given cos 120°=-1/2, use a half-angle formula to find the exact vertical height of the beam's endpoint above the mounting point.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Prove the identity sin(2x)/(1+cos(2x))=tan x. Use double-angle formulas and show each algebra step.

WORK SPACE AND ANSWER

ANSWER KEY

Exact Angle Toolkit

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The angle shown is θ. Use a double-angle formula to find the exact value of cos θ.	**-1/2**
2	Use an angle-sum formula to find the exact value of sin θ for the angle shown.	**1/2**
3	Use the tangent double-angle formula to find the exact value of tan 120°. Start with tan 60°=sqrt3.	**-sqrt3**
4	Given cos 105°=(sqrt2-sqrt6)/4, find the exact value of cos(105°/2). Note that 105°/2 is in Quadrant I.	**sqrt((4+sqrt2-sqrt6)/8)**
5	Use the sine angle-sum formula to simplify sin(x+x). Show the substitution and the simplified result.	**sin(x+x)=sin x cos x+cos x sin x=2 sin x cos x**
6	Solve for x, where 0°≤x≤90°: sin(x+90°)=1/2.	**x=60°**

PART 2 · APPLY IT

1. **-5 units** The horizontal change is 10 cos 120°. Since cos 120°=cos(2·60°)=-1/2, the change is -5 units.
2. **6sqrt3 m** The height is 12 sin 60°. Using sin 60°=sqrt((1-cos 120°)/2)=sqrt3/2 gives a height of 6sqrt3 m.

CHALLENGE

$\sin(2x)/(1+\cos(2x))=[2\sin x\cos x]/[1+(\cos^2 x-\sin^2 x)]=[2\sin x\cos x]/[2\cos^2 x]=\sin x/\cos x=\tan x$, when $\cos x\neq 0$.

Accept equivalent exact radical forms, including unsimplified forms that simplify to the listed answer. For the challenge, accept any valid use of $\sin(2x)=2\sin x\cos x$ and $1+\cos(2x)=2\cos^2 x$.