

Unit Circle Moves

Use coordinates to find exact trigonometric values.

HIGH SCHOOL • MA.912.T.2.2

MA.912.T.2.2 "Define the six basic trigonometric functions for all real numbers by identifying corresponding angle measures and using right triangles drawn in the unit circle."

NAME _____

DATE _____

COORDINATES GIVE THE FUNCTIONS

On the unit circle, the point at angle θ is (x, y) , where $\cos \theta = x$ and $\sin \theta = y$. Then $\tan \theta = y/x$, $\csc \theta = 1/y$, $\sec \theta = 1/x$, and $\cot \theta = x/y$. Use the reference angle and quadrant signs to find x and y , and write a function as undefined when its denominator is 0.

WORKED EXAMPLE

Find all six trigonometric functions for $\theta = 5\pi/4$.

- $\theta = 5\pi/4$ is in Quadrant III and has reference angle $\pi/4$.
- The unit-circle point is $(-\sqrt{2}/2, -\sqrt{2}/2)$.
- $\sin \theta = -\sqrt{2}/2$, $\cos \theta = -\sqrt{2}/2$, and $\tan \theta = 1$.
- Use reciprocals: $\csc \theta = -\sqrt{2}$, $\sec \theta = -\sqrt{2}$, and $\cot \theta = 1$.

Answer: $\sin \theta = -\sqrt{2}/2$, $\cos \theta = -\sqrt{2}/2$, $\tan \theta = 1$, $\csc \theta = -\sqrt{2}$, $\sec \theta = -\sqrt{2}$, $\cot \theta = 1$

PRACTICE 6 problems

- 1** Find $\sin \theta$ and $\cos \theta$ for $\theta = -\pi/3$.

ANSWER _____

- 2** Find $\tan \theta$ and $\cot \theta$ for $\theta = 7\pi/6$.

ANSWER _____

- 3** Find $\sec \theta$ and $\csc \theta$ for $\theta = 11\pi/6$.

ANSWER _____

- 4** A point on the unit circle is $(-1/2, -\sqrt{3}/2)$. Find an angle θ from 0 to 2π and find $\sin \theta$.

ANSWER _____

- 5** For $\theta = \pi$, identify each trigonometric function that is undefined.

ANSWER _____

- 6** For $\theta = \pi/2$, find $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$.

ANSWER _____

APPLY IT Show your work

- 1** A student codes a game character to move 24 units at an angle of $5\pi/6$ from the positive x-axis. What are the character's horizontal and vertical changes?

WORK SPACE

ANSWER _____

- 2** A theater light shines 10 meters from its starting point at an angle of $3\pi/4$ from the positive x-axis. How far left and how far up is the end of the light ray?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A point P is on the unit circle in Quadrant II, and $\tan \theta = -4/3$. Find all six trigonometric functions of θ . Explain how the quadrant determines the signs.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\sin \theta$ and $\cos \theta$ for $\theta = -\pi/3$.	$\sin \theta = -\sqrt{3}/2$, $\cos \theta = 1/2$
2	Find $\tan \theta$ and $\cot \theta$ for $\theta = 7\pi/6$.	$\tan \theta = \sqrt{3}/3$, $\cot \theta = \sqrt{3}$
3	Find $\sec \theta$ and $\csc \theta$ for $\theta = 11\pi/6$.	$\sec \theta = 2\sqrt{3}/3$, $\csc \theta = -2$
4	A point on the unit circle is $(-1/2, -\sqrt{3}/2)$. Find an angle θ from 0 to 2π and find $\sin \theta$.	$\theta = 4\pi/3$, $\sin \theta = -\sqrt{3}/2$
5	For $\theta = \pi$, identify each trigonometric function that is undefined.	$\csc \theta$ and $\cot \theta$ are undefined.
6	For $\theta = \pi/2$, find $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$.	$\sin \theta = 1$, $\cos \theta = 0$, $\tan \theta =$ undefined, $\csc \theta = 1$, $\sec \theta =$ undefined, $\cot \theta = 0$

PART 2 · APPLY IT

- Horizontal change: $-12\sqrt{3}$ units. Vertical change: 12 units.** Use $x = 24 \cos(5\pi/6)$ and $y = 24 \sin(5\pi/6)$. The unit-circle values are $\cos(5\pi/6) = -\sqrt{3}/2$ and $\sin(5\pi/6) = 1/2$.
- $5\sqrt{2}$ meters left and $5\sqrt{2}$ meters up.** The horizontal component is $10 \cos(3\pi/4) = -5\sqrt{2}$, so the light is $5\sqrt{2}$ meters left. The vertical component is $10 \sin(3\pi/4) = 5\sqrt{2}$ meters up.

CHALLENGE

$\sin \theta = 4/5$, $\cos \theta = -3/5$, $\tan \theta = -4/3$, $\csc \theta = 5/4$, $\sec \theta = -5/3$, $\cot \theta = -3/4$. A 3-4-5 reference triangle gives coordinate magnitudes $3/5$ and $4/5$ on the unit circle. In Quadrant II, x is negative and y is positive.

Accept equivalent radical forms, such as $1/\sqrt{3}$ for $\sqrt{3}/3$. Accept the word undefined for functions with a zero denominator.