

Circle Value Quest

Use special triangles to find exact trig values.

HIGH SCHOOL • MA.912.T.2.3

MA.912.T.2.3 "Determine the values of the six basic trigonometric functions for 0 , $\pi/6$, $\pi/3$ and $\pi/4$ and their multiples using special triangles."

NAME _____

DATE _____

SPECIAL TRIANGLE STRATEGY

For an angle on the unit circle, use a 30-60-90 or 45-45-90 reference triangle to find sine, cosine, and tangent, then use the quadrant to choose each sign. Cosecant, secant, and cotangent are the reciprocals of sine, cosine, and tangent.

WORKED EXAMPLE

Find all six trigonometric functions of $\pi/4$.

1. Use the 45-45-90 triangle ratio:
opposite = 1, adjacent = 1, hypotenuse = $\sqrt{2}$.
2. Find sine and cosine: $\sin(\pi/4) = 1/\sqrt{2} = \sqrt{2}/2$ and $\cos(\pi/4) = \sqrt{2}/2$.
3. Find tangent: $\tan(\pi/4) = 1/1 = 1$.
4. Take reciprocals: $\csc(\pi/4) = \sqrt{2}$, $\sec(\pi/4) = \sqrt{2}$, and $\cot(\pi/4) = 1$.

Answer: $\sin(\pi/4) = \sqrt{2}/2$, $\cos(\pi/4) = \sqrt{2}/2$, $\tan(\pi/4) = 1$, $\csc(\pi/4) = \sqrt{2}$, $\sec(\pi/4) = \sqrt{2}$, $\cot(\pi/4) = 1$

PRACTICE 6 problems

1 Find $\sin(\pi/6)$.

ANSWER _____

2 Find $\cos(3\pi/4)$.

ANSWER _____

3 Find $\tan(5\pi/6)$.

ANSWER _____

4 Find $\csc(4\pi/3)$.

ANSWER _____

5 Find $\sec(\pi/2)$.

ANSWER _____

6 Find $\cot(3\pi/2)$.

ANSWER _____

APPLY IT Show your work

- 1** At a school concert, a spotlight is 24 feet above the stage floor. Its beam makes an angle of $\pi/3$ with the floor and hits a performer. How long is the beam?

WORK SPACE

ANSWER _____

- 2** A design club plans a skateboard ramp that rises 6 feet at an angle of $\pi/6$ above level ground. What horizontal distance does the ramp cover?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $0 \leq \theta < 2\pi$, find all values of θ for which $\csc(\theta) = -2$. Explain how the reciprocal relationship and quadrant signs lead to your answers.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find $\sin(\pi/6)$.	$1/2$
2	Find $\cos(3\pi/4)$.	$-\sqrt{2}/2$
3	Find $\tan(5\pi/6)$.	$-\sqrt{3}/3$
4	Find $\csc(4\pi/3)$.	$-2\sqrt{3}/3$
5	Find $\sec(\pi/2)$.	undefined
6	Find $\cot(3\pi/2)$.	0

PART 2 • APPLY IT

- $16\sqrt{3}$ feet** The 24-foot height is opposite the $\pi/3$ angle, and the beam is the hypotenuse. Use $\sin(\pi/3) = 24/\text{beam length}$ to get a beam length of $16\sqrt{3}$ feet.
- $6\sqrt{3}$ feet** The rise is opposite the $\pi/6$ angle, and the horizontal distance is adjacent. Use $\tan(\pi/6) = 6/\text{horizontal distance}$ to get $6\sqrt{3}$ feet.

CHALLENGE

$\theta = 7\pi/6$ and $11\pi/6$. Since $\csc(\theta) = -2$, $\sin(\theta) = -1/2$. The reference angle is $\pi/6$, and sine is negative in Quadrants III and IV.

Accept equivalent exact forms, including unrationalized denominators when they are mathematically equivalent. For $\sec(\pi/2)$, accept undefined or does not exist.