

Turn and Reflect

Use unit-circle moves to find related trig values.

HIGH SCHOOL • MA.912.T.2.4

MA.912.T.2.4 "Use the unit circle to express the values of sine, cosine and tangent for $n-x$, $n+x$, and $2n-x$ in terms of their values for x , where x is any real number."

NAME _____

DATE _____

THREE ANGLE MOVES

On the unit circle, $\pi-x$ reflects x across the y -axis, $\pi+x$ adds a half-turn, and $2\pi-x$ reflects x across the x -axis. Therefore, $\sin(\pi-x)=\sin x$, $\cos(\pi-x)=-\cos x$, $\tan(\pi-x)=-\tan x$; $\sin(\pi+x)=-\sin x$, $\cos(\pi+x)=-\cos x$, $\tan(\pi+x)=\tan x$; and $\sin(2\pi-x)=-\sin x$, $\cos(2\pi-x)=\cos x$, $\tan(2\pi-x)=-\tan x$.

WORKED EXAMPLE

If $\cos x = 1/16$, find $\cos(\pi + x)$.

1. Use $\cos(\pi + x) = -\cos x$.
2. Substitute $\cos x = 1/16$.
3. $\cos(\pi + x) = -(1/16) = -1/16$.

Answer: $\cos(\pi + x) = -1/16$

PRACTICE 6 problems

1 If $\sin x = 3/5$, find $\sin(\pi - x)$.

ANSWER _____

2 If $\cos x = -4/9$, find $\cos(\pi + x)$.

ANSWER _____

3 If $\tan x = 5/12$, find $\tan(2\pi - x)$.

ANSWER _____

4 If $\sin x = -7/11$, find $\sin(\pi + x)$.

ANSWER _____

5 If $\cos x = 8/13$, find $\cos(2\pi - x)$.

ANSWER _____

6 If $\tan x = -9/14$, find $\tan(\pi - x)$.

ANSWER _____

APPLY IT Show your work

- 1** A music app models a repeating sound wave's vertical level with $\sin x$. At a new phase of $2\pi - x$, the app needs the vertical level. If $\sin x = -2/3$, what is $\sin(2\pi - x)$?

WORK SPACE

ANSWER _____

- 2** A game engine uses $\cos x$ to represent a character's horizontal direction value. After changing the direction angle to $\pi - x$, the original value was $\cos x = -3/8$. What is $\cos(\pi - x)$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For any real x for which $\tan x$ is defined, simplify and explain your reasoning: $[\sin(\pi - x)\cos(2\pi - x)]/[\cos(\pi + x)] + \tan(2\pi - x)$.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If $\sin x = 3/5$, find $\sin(\pi - x)$.	3/5
2	If $\cos x = -4/9$, find $\cos(\pi + x)$.	4/9
3	If $\tan x = 5/12$, find $\tan(2\pi - x)$.	-5/12
4	If $\sin x = -7/11$, find $\sin(\pi + x)$.	7/11
5	If $\cos x = 8/13$, find $\cos(2\pi - x)$.	8/13
6	If $\tan x = -9/14$, find $\tan(\pi - x)$.	9/14

PART 2 · APPLY IT

1. **2/3** Use $\sin(2\pi - x) = -\sin x$. The negative of $-2/3$ is $2/3$.
2. **3/8** Use $\cos(\pi - x) = -\cos x$. The negative of $-3/8$ is $3/8$.

CHALLENGE

$-\sin x - \tan x$. Since $\sin(\pi - x) = \sin x$, $\cos(2\pi - x) = \cos x$, $\cos(\pi + x) = -\cos x$, and $\tan(2\pi - x) = -\tan x$, the fraction becomes $-\sin x$ and the final term is $-\tan x$.

Accept equivalent simplified fractions with the correct sign. For the challenge, accept algebraically equivalent forms such as $-\sin x - \sin x / \cos x$, provided the student states that $\tan x$ is defined.