LESSON MAKER PRO

Unit Circle Flips

Track sine, cosine, and tangent through related angles.

HIGH SCHOOL · MA.912.T.2.4

MA.912.T.2.4 "Use the unit circle to express the values of sine, cosine and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number."

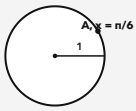
NAME _____ DATE _____

ANGLE TRANSFORMATIONS

On the unit circle, π-x reflects a point over the y-axis, π+x moves it to the opposite point, and 2π-x reflects it over the x-axis. For angles where tangent is defined, sin(π-x)=sin x, cos(π-x)=-cos x, and tan(π-x)=-tan x. Also, sin(π+x)=-sin x, cos(π+x)=-cos x, tan(π+x)=tan x; sin(2π-x)=-sin x, cos(2π-x)=cos x, and tan(2π-x)=-tan x.

WORKED EXAMPLE

In the unit-circle figure, point A marks x. Find sin(π+x).



1. Point A has coordinates (sqrt(3)/2, 1/2).
2. Adding π sends A to the opposite point, (-sqrt(3)/2, -1/2).
3. Sine is the y-coordinate, so sin(π+x)=-1/2.

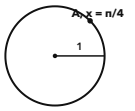
Answer: -1/2

PRACTICE 6 problems

1 If tan x=-7/2, find tan(π-x).

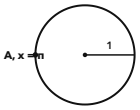
ANSWER _____

2 In the unit-circle figure, point A marks x. Find cos(π+x).



ANSWER _____

3 In the unit-circle figure, point A marks x. Find sin(2π-x).



ANSWER _____

4 If cos x=-5/13, find cos(2π-x).

ANSWER _____

5 If sin x=-8/17, find sin(π+x).

ANSWER _____

6 If tan x=3/4, find tan(π+x).

ANSWER _____

APPLY IT Show your work

1 In a drone-racing game, a camera's aim is angle x. The game records sin x=3/5 and cos x=-4/5. A replay changes the aim to π-x. What are sin(π-x) and tan(π-x)?

WORK SPACE

ANSWER _____

2 In a music app, an icon is at unit-circle angle x with coordinates (8/17, -15/17). The app reverses the direction using 2π-x. What are cos(2π-x) and tan(2π-x)?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For any x for which tangent is defined, explain why tan(π-x)=-tan x. Use the sine and cosine relationships. Also state what happens if tan x is undefined.

WORK SPACE AND ANSWER

ANSWER KEY

Unit Circle Flips

HIGH SCHOOL · MA.912.T.2.4

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If tan x=-7/2, find tan(π-x).	**7/2**
2	In the unit-circle figure, point A marks x. Find cos(π+x).	**-sqrt(2)/2**
3	In the unit-circle figure, point A marks x. Find sin(2π-x).	**0**
4	If cos x=-5/13, find cos(2π-x).	**-5/13**
5	If sin x=-8/17, find sin(π+x).	**8/17**
6	If tan x=3/4, find tan(π+x).	**3/4**

PART 2 · APPLY IT

1. **sin(π-x)=3/5; tan(π-x)=3/4** Use sin(π-x)=sin x to get 3/5. Since tan x=(3/5)/(-4/5)=-3/4, tan(π-x)=-tan x=3/4.
2. **cos(2π-x)=8/17; tan(2π-x)=15/8** Cosine stays the same under 2π-x, so it is 8/17. Sine changes to 15/17, and tangent is (15/17)/(8/17)=15/8.

CHALLENGE

tan(π-x)=sin(π-x)/cos(π-x)=sin x/(-cos x)=-tan x. If tan x is undefined because cos x=0, then cos(π-x)=0 too, so tan(π-x) is undefined.

Accept equivalent radical forms, such as -sqrt(2)/2 or -1/sqrt(2). For the challenge, accept any explanation that correctly uses the sine and cosine transformations and identifies the undefined case.