

Unit Circle Toolkit

Find six exact trig values from one angle.

HIGH SCHOOL • MA.912.T.2.5

MA.912.T.2.5 "Given angles measured in radians or degrees, calculate the values of the six basic trigonometric functions using the unit circle, trigonometric identities or technology."

NAME _____

DATE _____

COORDINATES MAKE VALUES

On the unit circle, the terminal point for an angle θ is $(\cos \theta, \sin \theta)$. Use $\tan \theta = \sin \theta / \cos \theta$, then use reciprocal pairs: $\csc$ and $\sin$, $\sec$ and $\cos$, $\cot$ and $\tan$. If a denominator is 0, that trig value is undefined.

WORKED EXAMPLE

Find $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$ when $\theta = \pi/4$.

1. The terminal point for $\pi/4$ is $(\sqrt{2}/2, \sqrt{2}/2)$.
2. $\sin \theta = \sqrt{2}/2$ and $\cos \theta = \sqrt{2}/2$.
3. $\tan \theta = (\sqrt{2}/2)/(\sqrt{2}/2) = 1$.
4. Use reciprocals: $\csc \theta = \sqrt{2}$, $\sec \theta = \sqrt{2}$, and $\cot \theta = 1$.

Answer: $\sin \theta = \sqrt{2}/2$, $\cos \theta = \sqrt{2}/2$, $\tan \theta = 1$, $\csc \theta = \sqrt{2}$, $\sec \theta = \sqrt{2}$, $\cot \theta = 1$

PRACTICE 6 problems

1 Find all six trig values when $\theta = 0$.

ANSWER _____

2 Find all six trig values when $\theta = \pi/6$.

ANSWER _____

3 Find all six trig values when $\theta = 3\pi/4$.

ANSWER _____

4 Find all six trig values when $\theta = 210$ degrees.

ANSWER _____

5 Find all six trig values when $\theta = 11\pi/6$.

ANSWER _____

6 Find all six trig values when $\theta = 270$ degrees.

ANSWER _____

APPLY IT Show your work

- 1** An animation student rotates a character's direction by 120 degrees from the positive x-axis. The animation app represents the direction with the unit-circle point $(\cos \theta, \sin \theta)$. Find all six trig values for $\theta = 120$ degrees.

WORK SPACE

ANSWER _____

- 2** A game designer places a virtual spotlight at an angle of $5\pi/3$ on a circular map. The map uses the unit circle, so the spotlight direction is $(\cos \theta, \sin \theta)$. Find all six trig values for $\theta = 5\pi/3$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

An angle θ has $\sec \theta = -2$, and its terminal side is in Quadrant III. Find all six trig values. Explain how the quadrant helps you choose the signs.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find all six trig values when $\theta = 0$.	$\sin \theta = 0$, $\cos \theta = 1$, $\tan \theta = 0$, $\csc \theta = \text{undefined}$, $\sec \theta = 1$, $\cot \theta = \text{undefined}$
2	Find all six trig values when $\theta = \pi/6$.	$\sin \theta = 1/2$, $\cos \theta = \sqrt{3}/2$, $\tan \theta = \sqrt{3}/3$, $\csc \theta = 2$, $\sec \theta = 2\sqrt{3}/3$, $\cot \theta = \sqrt{3}$
3	Find all six trig values when $\theta = 3\pi/4$.	$\sin \theta = \sqrt{2}/2$, $\cos \theta = -\sqrt{2}/2$, $\tan \theta = -1$, $\csc \theta = \sqrt{2}$, $\sec \theta = -\sqrt{2}$, $\cot \theta = -1$
4	Find all six trig values when $\theta = 210$ degrees.	$\sin \theta = -1/2$, $\cos \theta = -\sqrt{3}/2$, $\tan \theta = \sqrt{3}/3$, $\csc \theta = -2$, $\sec \theta = -2\sqrt{3}/3$, $\cot \theta = \sqrt{3}$
5	Find all six trig values when $\theta = 11\pi/6$.	$\sin \theta = -1/2$, $\cos \theta = \sqrt{3}/2$, $\tan \theta = -\sqrt{3}/3$, $\csc \theta = -2$, $\sec \theta = 2\sqrt{3}/3$, $\cot \theta = -\sqrt{3}$
6	Find all six trig values when $\theta = 270$ degrees.	$\sin \theta = -1$, $\cos \theta = 0$, $\tan \theta = \text{undefined}$, $\csc \theta = -1$, $\sec \theta = \text{undefined}$, $\cot \theta = 0$

PART 2 · APPLY IT

- $\sin \theta = \sqrt{3}/2$, $\cos \theta = -1/2$, $\tan \theta = -\sqrt{3}$, $\csc \theta = 2\sqrt{3}/3$, $\sec \theta = -2$, $\cot \theta = -\sqrt{3}/3$** Use the 60-degree reference angle in Quadrant II, where sine is positive and cosine is negative. Find tangent by dividing sine by cosine, then use reciprocals.
- $\sin \theta = -\sqrt{3}/2$, $\cos \theta = 1/2$, $\tan \theta = -\sqrt{3}$, $\csc \theta = -2\sqrt{3}/3$, $\sec \theta = 2$, $\cot \theta = -\sqrt{3}/3$** Use the $\pi/3$ reference angle in Quadrant IV, where sine is negative and cosine is positive. Divide to find tangent and take reciprocals for csc, sec, and cot.

CHALLENGE

$\cos \theta = -1/2$ because $\sec \theta = -2$. In Quadrant III, $\sin \theta$ is negative, so $\sin \theta = -\sqrt{3}/2$. Therefore $\tan \theta = \sqrt{3}$, $\csc \theta = -2\sqrt{3}/3$, $\sec \theta = -2$, and $\cot \theta = \sqrt{3}/3$.

Accept equivalent exact forms, including rationalized or unrationalized denominators. Require the word undefined for values with a zero denominator.