

Circle Value Quest

Use unit-circle coordinates to find exact trig values.

HIGH SCHOOL • MA.912.T.2

MA.912.T.2 "Extend trigonometric functions to the unit circle."

NAME _____

DATE _____

UNIT CIRCLE RULES

On the unit circle, the point at angle theta is (cos theta, sin theta), so $\tan \theta = \sin \theta / \cos \theta$. Use the reference angle to find the basic value, then use the quadrant to choose its sign. Secant, cosecant, and cotangent are reciprocal functions.

WORKED EXAMPLE

Find $\sin(7\pi/6)$.

1. $7\pi/6$ is in Quadrant III, and its reference angle is $\pi/6$.
2. Sine is negative in Quadrant III.
3. $\sin(\pi/6) = 1/2$, so $\sin(7\pi/6) = -1/2$.

Answer: -1/2

PRACTICE 6 problems

- 1 Write the unit-circle coordinates for $\theta = \pi/4$.

ANSWER _____

- 2 Find $\cos(3\pi/4)$.

ANSWER _____

- 3 Find $\tan(5\pi/3)$.

ANSWER _____

- 4 Find $\sec(2\pi/3)$.

ANSWER _____

- 5 Find $\csc(4\pi/3)$.

ANSWER _____

- 6 Find $\cot(\pi/3)$.

ANSWER _____

APPLY IT Show your work

- 1** A robotics team programs a camera arm that is 10 feet long. The arm starts pointing right from its pivot and turns counterclockwise $5\pi/4$. What are the coordinates of the camera relative to the pivot?

WORK SPACE

ANSWER _____

- 2** A music app has a visualizer dot 9 units from the center of its circular display. The dot starts at the right side and moves clockwise $\pi/3$. What are its coordinates relative to the center?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $0 \leq \theta < 2\pi$, find all angles where $\sin \theta = \cos \theta$. Explain how the unit circle helps you find both angles.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write the unit-circle coordinates for $\theta = \pi/4$.	$(\sqrt{2}/2, \sqrt{2}/2)$
2	Find $\cos(3\pi/4)$.	$-\sqrt{2}/2$
3	Find $\tan(5\pi/3)$.	$-\sqrt{3}$
4	Find $\sec(2\pi/3)$.	-2
5	Find $\csc(4\pi/3)$.	$-2\sqrt{3}/3$
6	Find $\cot(\pi/3)$.	$\sqrt{3}/3$

PART 2 · APPLY IT

1. **$(-\sqrt{2}, -\sqrt{2})$** Use $x = 10\cos(5\pi/4)$ and $y = 10\sin(5\pi/4)$. Both unit-circle values are $-\sqrt{2}/2$.
2. **$(9/2, -9\sqrt{3}/2)$** Moving clockwise $\pi/3$ gives an angle of $-\pi/3$. Use $x = 9\cos(-\pi/3)$ and $y = 9\sin(-\pi/3)$.

CHALLENGE

$\theta = \pi/4$ or $5\pi/4$. On the unit circle, sine and cosine are equal when the y-coordinate equals the x-coordinate, which happens in Quadrants I and III.

Accept equivalent exact radical forms, including $-2/\sqrt{3}$ for $\csc(4\pi/3)$. For the challenge, require both angles and reasoning that equal sine and cosine values mean equal y- and x-coordinates.