

Wave Tracker

Model, sketch, and solve with sine and cosine.

HIGH SCHOOL • MA.912.T.3.3

MA.912.T.3.3 "Solve and graph mathematical and real-world problems that are modeled with trigonometric functions. Interpret key features and determine constraints in terms of the context."

NAME _____

DATE _____

KEY FEATURES OF A WAVE

For $y = A \sin(Bx) + D$ or $y = A \cos(Bx) + D$, the amplitude is $|A|$, the period is $2\pi/|B|$, and the midline is $y = D$. The range goes from $D - |A|$ to $D + |A|$, and a negative A reflects the basic curve across its midline.

WORKED EXAMPLE

Sketch one cycle of $y = 9 \sin(4x) + 5$ by listing its five key points.

1. Amplitude = $|9| = 9$, midline: $y = 5$.
2. Period = $2\pi/4 = \pi/2$.
3. Divide one period into four equal parts:
 $0, \pi/8, \pi/4, 3\pi/8, \pi/2$.
4. Use sine values $0, 1, 0, -1, 0$ to find the y -values.

Answer: $(0, 5)$, $(\pi/8, 14)$, $(\pi/4, 5)$, $(3\pi/8, -4)$, $(\pi/2, 5)$

PRACTICE 6 problems

- 1** Sketch one cycle of $y = 8 \sin(x) - 2$ by listing its five key points from $x = 0$ to $x = 2\pi$.

ANSWER _____

- 2** For $y = -6 \cos(x) + 3$, state the amplitude, midline, range, and whether the graph is reflected across its midline.

ANSWER _____

- 3** Solve $2 \sin(x) = \sqrt{3}$ for $0 \leq x < 2\pi$.

ANSWER _____

- 4** Solve $\cos(3x) = 0$ for $0 \leq x < 2\pi$.

ANSWER _____

- 5** Sketch one cycle of $y = 7 \sin(\pi x)$ by listing its five key points.

ANSWER _____

- 6** For $y = 10 \cos(x/2) - 1$, state the amplitude, period, midline, and range.

ANSWER _____

APPLY IT Show your work

- 1** At a school festival, a Ferris wheel rider's height H , in meters, is modeled by $H(t) = 14 - 8 \cos(\pi t/6)$, where t is time in minutes and $0 \leq t \leq 12$. Find the minimum and maximum heights. Then find every time when the rider is 14 meters above the ground.

WORK SPACE

ANSWER _____

- 2** A concert light display has brightness $B(t) = 40 + 30 \sin(\pi t/8)$, where t is measured in seconds and $0 \leq t \leq 16$. Find the period, the greatest and least brightness, and the times when each occurs. List five key points that could be used to sketch the display.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A ride's height has a minimum of 6 meters and a maximum of 18 meters. It completes one cycle every 12 seconds and starts at its maximum height when $t = 0$. Write a cosine model $h(t)$. Then find every time the ride is at its midline height during $0 \leq t \leq 24$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Sketch one cycle of $y = 8 \sin(x) - 2$ by listing its five key points from $x = 0$ to $x = 2\pi$.	(0, -2), ($\pi/2$, 6), (π, -2), ($3\pi/2$, -10), (2π, -2)
2	For $y = -6 \cos(x) + 3$, state the amplitude, midline, range, and whether the graph is reflected across its midline.	Amplitude: 6; midline: $y = 3$; range: $-3 \leq y \leq 9$; reflected across the midline.
3	Solve $2 \sin(x) = \sqrt{3}$ for $0 \leq x < 2\pi$.	$x = \pi/3, 2\pi/3$
4	Solve $\cos(3x) = 0$ for $0 \leq x < 2\pi$.	$x = \pi/6, \pi/2, 5\pi/6, 7\pi/6, 3\pi/2, 11\pi/6$
5	Sketch one cycle of $y = 7 \sin(\pi x)$ by listing its five key points.	(0, 0), (1/2, 7), (1, 0), (3/2, -7), (2, 0)
6	For $y = 10 \cos(x/2) - 1$, state the amplitude, period, midline, and range.	Amplitude: 10; period: 4π; midline: $y = -1$; range: $-11 \leq y \leq 9$

PART 2 • APPLY IT

- Minimum height: 6 meters at $t = 0$ and $t = 12$. Maximum height: 22 meters at $t = 6$. The rider is 14 meters high at $t = 3$ and $t = 9$.** The midline is 14 and the amplitude is 8, so the heights range from 6 to 22 meters. Set $\cos(\pi t/6) = 0$ to get $t = 3$ and $t = 9$ in the stated domain.
- Period: 16 seconds. Greatest brightness: 70 at $t = 4$. Least brightness: 10 at $t = 12$. Key points: (0, 40), (4, 70), (8, 40), (12, 10), (16, 40).** The period is $2\pi/(\pi/8) = 16$, and the brightness is 40 plus or minus 30. Use the quarter-period times 0, 4, 8, 12, and 16 to locate the key points.

CHALLENGE

$h(t) = 12 + 6 \cos(\pi t/6)$. The midline height is 12 meters, reached at $t = 3, 9, 15$, and 21 seconds.

Accept equivalent ordered pairs, equivalent angle forms, and equivalent interval notation for ranges. For graphing items, accept a smooth sinusoidal curve through the listed key points.