

# Polar Path Sketches

*Turn polar equations into clear graphs.*

**HIGH SCHOOL • MA.912.T.4.3**

**MA.912.T.4.3** "Graph equations in the polar coordinate plane with and without the use of graphing technology."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## CIRCLE AND LINE CLUES

To graph a polar equation, use  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ , and  $r^2 = x^2 + y^2$  when a rectangular form is easier to sketch. For an equation with  $r$  and sine or cosine, multiply both sides by  $r$  first, then complete the square if needed.

## WORKED EXAMPLE

Graph  $r = 10\cos(\theta)$ . State the center and radius.

1. Multiply both sides by  $r$ :  $r^2 = 10r \cos(\theta)$ .
2. Substitute rectangular expressions:  $x^2 + y^2 = 10x$ .
3. Complete the square:  $(x - 5)^2 + y^2 = 25$ .
4. Recognize a circle from the standard form.

**Answer: Circle with center (5, 0) and radius 5.**

## PRACTICE 6 problems

- 1** Graph  $r = 2\cos(\theta)$ . State the center and radius.

**ANSWER** \_\_\_\_\_

- 2** Graph  $r = 6\sin(\theta)$ . State the center and radius.

**ANSWER** \_\_\_\_\_

- 3** Graph  $r = -8\cos(\theta)$ . State the center and radius.

**ANSWER** \_\_\_\_\_

- 4** Graph  $r = 3\sec(\theta)$ . State its rectangular equation and identify the line.

**ANSWER** \_\_\_\_\_

- 5** Graph  $r = 4\csc(\theta)$ . State its rectangular equation and identify the line.

**ANSWER** \_\_\_\_\_

- 6** Graph  $r = 2\sin(2\theta)$ . State the number of petals, the length of each petal, and the directions of the petal tips.

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** The art club uses a polar map with the school entrance at the pole. A circular chalk mural has boundary  $r = 8\cos(\theta)$ . Graph the boundary, then state the center and radius of the mural.

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A concert crew maps a straight row of light panels using polar coordinates with the stage at the pole. The row is modeled by  $r = 6\csc(\theta)$ . Graph the row and write its rectangular equation.

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Graph  $r = 4\cos(\theta) + 6\sin(\theta)$ . State the center and radius, then explain how you know the graph is a circle.

---

---

## PART 1 · PRACTICE

| # | PROBLEM                                                                                                                  | ANSWER                                                                                                                                                  |
|---|--------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | Graph $r = 2\cos(\theta)$ . State the center and radius.                                                                 | <b>Circle with center (1, 0) and radius 1.</b>                                                                                                          |
| 2 | Graph $r = 6\sin(\theta)$ . State the center and radius.                                                                 | <b>Circle with center (0, 3) and radius 3.</b>                                                                                                          |
| 3 | Graph $r = -8\cos(\theta)$ . State the center and radius.                                                                | <b>Circle with center (-4, 0) and radius 4.</b>                                                                                                         |
| 4 | Graph $r = 3\sec(\theta)$ . State its rectangular equation and identify the line.                                        | <b><math>x = 3</math>, a vertical line.</b>                                                                                                             |
| 5 | Graph $r = 4\csc(\theta)$ . State its rectangular equation and identify the line.                                        | <b><math>y = 4</math>, a horizontal line.</b>                                                                                                           |
| 6 | Graph $r = 2\sin(2\theta)$ . State the number of petals, the length of each petal, and the directions of the petal tips. | <b>4 petals, each with length 2, with tips in the directions <math>\pi/4</math>, <math>3\pi/4</math>, <math>5\pi/4</math>, and <math>7\pi/4</math>.</b> |

## PART 2 · APPLY IT

- The mural is a circle with center (4, 0) and radius 4.** Multiply by  $r$  to get  $x^2 + y^2 = 8x$ . Completing the square gives  $(x - 4)^2 + y^2 = 16$ .
- $y = 6$ , a horizontal line.** Multiply by  $\sin(\theta)$  to get  $r \sin(\theta) = 6$ . Since  $r \sin(\theta) = y$ , the graph is  $y = 6$ .

## CHALLENGE

**It is a circle with center (2, 3) and radius  $\sqrt{13}$ .** Multiplying by  $r$  gives  $x^2 + y^2 = 4x + 6y$ , so  $(x - 2)^2 + (y - 3)^2 = 13$ .

Accept equivalent standard-form equations and equivalent descriptions of the same circle or line. For the rose curve, accept petal directions that use negative angles or coterminal angles.