

Polar Curve Studio

Recognize special polar graphs from their equations.

HIGH SCHOOL • MA.912.T.4.4

MA.912.T.4.4 "Identify and graph special polar equations, including circles, cardioids, limacons, rose curves and lemniscates."

NAME _____

DATE _____

SPOT THE PATTERN

For $r = a \cos(\theta)$ or $r = a \sin(\theta)$, the graph is a circle. Equations of the form $r = a \pm b \cos(\theta)$ or $r = a \pm b \sin(\theta)$ make cardioids when $a = b$ and limacons when $a \neq b$, while $r = a \cos(n\theta)$ or $r = a \sin(n\theta)$ makes a rose with n petals if n is odd and $2n$ petals if n is even. Equations such as $r^2 = a^2 \cos(2\theta)$ or $r^2 = a^2 \sin(2\theta)$ make two-loop lemniscates.

WORKED EXAMPLE

Graph $r = 8 \cos(7\theta)$. Identify the curve and state its petal count and petal length.

1. The equation has the form $r = a \cos(n\theta)$, so it is a rose curve.
2. Here $n = 7$, which is odd, so the graph has 7 petals.
3. Each petal has length $|8|$, so each petal is 8 units long.
4. The petal tips lie on the rays $\theta = 2k\pi/7$ for $k = 0$ through 6.

Answer: Rose curve with 7 petals, each 8 units long.

PRACTICE 6 problems

- 1** Identify and graph $r = 3 \sin(\theta)$. State the circle's center and radius.

ANSWER _____

- 2** Identify and graph $r = 1 + \cos(\theta)$. State the cusp, opening direction, and farthest point.

ANSWER _____

- 3** Identify and graph $r = 2 + 4 \cos(\theta)$. State whether it has an inner loop and give its pole crossings.

ANSWER _____

- 4** Identify and graph $r = 2 \sin(3\theta)$. State the number of petals, petal length, and petal-center angles.

ANSWER _____

- 5** Identify and graph $r = 3 \cos(2\theta)$. State the number of petals, petal length, and their directions.

ANSWER _____

- 6** Identify and graph $r^2 = 9 \sin(2\theta)$. State the loop length and the directions of the loops.

ANSWER _____

APPLY IT Show your work

- 1** A student designs a profile image with the polar boundary $r = 2 - 2 \sin(\theta)$. Identify the curve and state its opening direction, cusp location, and farthest point.

WORK SPACE

ANSWER _____

- 2** A robotics team sets a drone's safety boundary with $r = 6 \cos(\theta)$, where the launch point is the pole and the positive polar axis points east. What shape is the boundary? Give its center and radius.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Graph both $r^2 = 4 \cos(2\theta)$ and $r^2 = -4 \cos(2\theta)$. Explain how the two graphs are alike and how they differ.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Identify and graph $r = 3 \sin(\theta)$. State the circle's center and radius.	Circle. Center $(0, 3/2)$, radius $3/2$. It passes through the pole and is symmetric about the y-axis.
2	Identify and graph $r = 1 + \cos(\theta)$. State the cusp, opening direction, and farthest point.	Cardioid. It has a cusp at the pole, opens right, and has farthest point $(2, 0)$.
3	Identify and graph $r = 2 + 4 \cos(\theta)$. State whether it has an inner loop and give its pole crossings.	Limaçon with an inner loop. It is symmetric about the polar axis, has an outer tip at $(6, 0)$, and crosses the pole at $\theta = 2\pi/3$ and $\theta = 4\pi/3$. The inner loop is on the right.
4	Identify and graph $r = 2 \sin(3\theta)$. State the number of petals, petal length, and petal-center angles.	Rose curve with 3 petals, each 2 units long. The petal centers are at $\theta = \pi/6, 5\pi/6$, and $3\pi/2$.
5	Identify and graph $r = 3 \cos(2\theta)$. State the number of petals, petal length, and their directions.	Rose curve with 4 petals, each 3 units long. The petals lie along the positive and negative x-axis and y-axis.
6	Identify and graph $r^2 = 9 \sin(2\theta)$. State the loop length and the directions of the loops.	Lemniscate with 2 loops, each reaching 3 units from the pole. The loops point along $\theta = \pi/4$ and $\theta = 5\pi/4$.

PART 2 • APPLY IT

- Cardioid. It opens downward, has a cusp at the pole, and has farthest point $(0, -4)$.** Because the sine term is subtracted, the largest radius occurs at $\theta = 3\pi/2$. When $\theta = \pi/2$, $r = 0$, so the cusp is at the pole.
- The boundary is a circle with center $(3, 0)$ and radius 3.** Multiply by r to get $r^2 = 6r \cos(\theta)$, or $x^2 + y^2 = 6x$. Completing the square gives $(x - 3)^2 + y^2 = 9$.

CHALLENGE

Both graphs are lemniscates with 2 loops, and each loop reaches 2 units from the pole. $r^2 = 4 \cos(2\theta)$ has horizontal loops along the x-axis, while $r^2 = -4 \cos(2\theta)$ has vertical loops along the y-axis. The second graph is the first graph rotated 90 degrees.

Accept equivalent angle labels that locate the same rays, such as angles differing by 2π . Accept accurate sketches with the correct symmetry, orientation, pole crossings, and key lengths.