

Coordinate Switch

Move between rectangular, polar, and parametric forms.

HIGH SCHOOL • MA.912.T.4

MA.912.T.4 "Extend rectangular coordinates and equations to polar and parametric forms."

NAME _____

DATE _____

SWITCHING COORDINATE FORMS

Polar coordinates locate a point with r , its distance from the origin, and θ , its angle from the positive x -axis. Use $r = \sqrt{x^2 + y^2}$, $x = r \cos(\theta)$, and $y = r \sin(\theta)$, with $r \geq 0$ and $0^\circ \leq \theta < 360^\circ$. In parametric form, write both x and y using a parameter such as t to show how a point moves.

WORKED EXAMPLE

Write the rectangular point $(8, 15)$ in polar form.

- $r = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$.
- The point is in Quadrant I, so $\theta = \tan^{-1}(15/8)$.
- The polar coordinates are $(17, \tan^{-1}(15/8))$.

Answer: $(17, \tan^{-1}(15/8))$

PRACTICE 6 problems

- 1** Write the rectangular point $(3, 4)$ in polar form.

ANSWER _____

- 2** Write the rectangular point $(-4, 3)$ in polar form.

ANSWER _____

- 3** Write the polar coordinates $(6, 30^\circ)$ in rectangular form.

ANSWER _____

- 4** Convert $x^2 + y^2 = 81$ to polar form.

ANSWER _____

- 5** Eliminate t from $x = 1 + 2t$ and $y = 5 - 3t$.

ANSWER _____

- 6** A moving point starts at $(2, -1)$. Each second, it moves 4 units right and 3 units down. Let t be time in seconds. Write parametric equations for the motion.

ANSWER _____

APPLY IT Show your work

- 1** In a racing game, a player token starts at $(-2,6)$. Each frame, it moves 3 units right and 2 units down. Let t be the number of frames. Write parametric equations for the token, then find its location after 4 frames.

WORK SPACE

ANSWER _____

- 2** On a music festival map, the main gate is the origin. A food truck is located at polar coordinates $(12,150^\circ)$, where the positive x -axis points east and the positive y -axis points north. Write the truck's rectangular coordinates.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A motion is modeled by $x=3t$ and $y=4t$. Describe the polar coordinates of the point when $t>0$ and when $t<0$. Explain why the angle changes.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write the rectangular point (3,4) in polar form.	$(5, \tan^{-1}(4/3))$
2	Write the rectangular point (-4,3) in polar form.	$(5, 180^\circ - \tan^{-1}(3/4))$
3	Write the polar coordinates (6,30°) in rectangular form.	$(3\sqrt{3}, 3)$
4	Convert $x^2 + y^2 = 81$ to polar form.	$r = 9$
5	Eliminate t from $x = 1 + 2t$ and $y = 5 - 3t$.	$3x + 2y = 13$
6	A moving point starts at (2,-1). Each second, it moves 4 units right and 3 units down. Let t be time in seconds. Write parametric equations for the motion.	$x = 2 + 4t, y = -1 - 3t, t \geq 0$

PART 2 · APPLY IT

1. $x = -2 + 3t, y = 6 - 2t$; after 4 frames, (10,-2) Use the starting coordinates as the constants and the movement per frame as the coefficients of t. Substitute $t = 4$ to find the location.
2. $(-6\sqrt{3}, 6)$ Use $x = r \cos(\theta)$ and $y = r \sin(\theta)$. Since $\cos(150^\circ) = -\sqrt{3}/2$ and $\sin(150^\circ) = 1/2$, the coordinates are $(-6\sqrt{3}, 6)$.

CHALLENGE

For $t > 0$, $(r, \theta) = (5t, \tan^{-1}(4/3))$. For $t < 0$, $(r, \theta) = (5|t|, 180^\circ + \tan^{-1}(4/3))$. When t changes from positive to negative, both x and y change signs, so the point moves in the opposite direction and the angle increases by 180° . At $t = 0$, the point is at the origin and theta is undefined.

Accept coterminal angles that differ by multiples of 360° . Accept equivalent exact inverse-trigonometric notation and algebraically equivalent rectangular equations.