

Squares Tell Stories

Use area models to test right triangles.

GRADE 8 • 8.GSR.8.1

8.GSR.8.1 "Explain a proof of the Pythagorean Theorem and its converse using visual models."

NAME _____

DATE _____

SEE THE SQUARES

Study this area model, a large square with side $a+b$, packed with four congruent right triangles and a center square: +-----
-----+ | triangle a,b,c | | triangle
 a,b,c [center square c by c] triangle a,b,c | | triangle a,b,c | +----
-----+ The big square has area $(a+b)^2$, while its pieces have area $4(ab/2)+c^2$, so matching the areas gives $a^2+b^2=c^2$. For the converse, build the right triangle shown with legs a and b , then a triangle with the same three side lengths and $c^2=a^2+b^2$ must match it, so the angle opposite c is right.

WORKED EXAMPLE

Use the area model to prove the relationship for a right triangle with legs 8 and 15 and hypotenuse c .

1. Place four copies of the 8-by-15 right triangle in a large square with side $8+15=23$.
2. Find the large-square area: $23^2=529$.
3. Find the area of four triangles:
 $4(8 \cdot 15/2)=240$.
4. The center-square area is $529-240=289=17^2$, so $c=17$.

Answer: $c=17$

PRACTICE 6 problems

- 1** A right triangle has legs 7 cm and 24 cm. Find the hypotenuse c .

ANSWER _____

- 2** A right triangle has hypotenuse 26 and one leg 10. Find x , the other leg.

ANSWER _____

- 3** Can a triangle with side lengths 12, 35, and 37 be a right triangle?

ANSWER _____

- 4** Can a triangle with side lengths 12, 16, and 21 be a right triangle?

ANSWER _____

- 5** A right triangle has hypotenuse 29 and one leg 20. What is the area of the square built on the other leg?

ANSWER _____

- 6** A triangle has side lengths 11, 60, and 61. Which side is the hypotenuse, and is the triangle right?

ANSWER _____

APPLY IT Show your work

- 1** A gaming club uses a rectangular projection screen that is 18 feet wide and 24 feet tall. What is the length of the screen's diagonal?

WORK SPACE

ANSWER _____

- 2** A skateboard ramp rises 20 feet over a horizontal distance of 21 feet. How long is the ramp surface?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

In exactly two sentences, use the visual area model and the converse to explain why a triangle with side lengths 28, 45, and 53 is a right triangle.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A right triangle has legs 7 cm and 24 cm. Find the hypotenuse c .	$c=25$ cm
2	A right triangle has hypotenuse 26 and one leg 10. Find x , the other leg.	$x=24$
3	Can a triangle with side lengths 12, 35, and 37 be a right triangle?	Yes, because $12^2+35^2=37^2$.
4	Can a triangle with side lengths 12, 16, and 21 be a right triangle?	No, because $12^2+16^2=400$ and $21^2=441$.
5	A right triangle has hypotenuse 29 and one leg 20. What is the area of the square built on the other leg?	441 square units
6	A triangle has side lengths 11, 60, and 61. Which side is the hypotenuse, and is the triangle right?	61 is the hypotenuse, and the triangle is right.

PART 2 • APPLY IT

- 30 feet** Use $18^2+24^2=324+576=900$. Since $900=30^2$, the diagonal is 30 feet.
- 29 feet** Use $20^2+21^2=400+441=841$. Since $841=29^2$, the ramp surface is 29 feet long.

CHALLENGE

Since $28^2+45^2=784+2025=2809=53^2$, the side lengths pass the converse test. In the area model, a right triangle with legs 28 and 45 has a center square with side 53, and matching side lengths mean the given triangle has a right angle opposite 53.

Accept equivalent equations and correct unit labels. For the challenge, accept any two-sentence explanation that verifies the squared-length relationship and connects the matching side lengths to the visual model or converse.