Fountain Flow

Predict water speed and pressure inside a changing pipe.

HS-PSII-3.4 "Develop and apply Bernoulli's principle and continuity equations to predict changes in the speed and pressure of a moving incompressible fluid."

NAME _____ DATE _____

1 READ AND MARK

Read it twice

YOUR JOB Underline the two equations, circle the conditions needed to use them, and number the three pipe sections in flow order.

Inside the Fountain Pipe

1. A city fountain sends water through a level pipe before it sprays upward. Focus on the part of the pipe that stays at the same height, before the outlet. Imagine steady flow with no leaks. Water is nearly incompressible, so the volume passing any cross section each second is the same. If a section has area A and average speed v, its flow rate is Av. At two sections, $A_1v_1 = A_2v_2$. A smaller area therefore requires a higher speed; a larger area requires a lower speed. These predictions concern water inside the pipe, not its motion after it leaves.

2. Speed alone does not describe what the water does. Static pressure measures the push water exerts on the pipe wall. For steady flow along a level pipe, with negligible friction and no pump between sections, Bernoulli's principle gives $P + (1/2)\rho v^2 = \text{constant}$. Here P is static pressure, ρ is density, and v is speed. When speed rises, static pressure falls; when speed falls, static pressure rises. The equation connects speed and pressure at different sections, but only under its stated conditions. It does not say that every fast-moving fluid everywhere has low pressure.

3. In the fountain's straight, horizontal supply pipe, sensors mark three sections: A before a constriction, B within the constriction, and C after the pipe widens. Their cross-sectional areas are 8 cm^2, 4 cm^2, and 16 cm^2, respectively. At A, the water travels at 2 m/s and has a static pressure of 120,000 Pa. Treat the flow as steady and the pressure loss from friction as negligible. Use a water density of 1,000 kg/m^3. Compare sections using the same flow rate, then use the change in speed to find each pressure. The pipe stays level through all three sections.

2 ANSWER WITH EVIDENCE

Use the passage

1. Use the continuity equation to find the water's speed at B. Show your calculation.

2. Use the continuity equation to find the water's speed at C. Show your calculation.

3. Use Bernoulli's equation to find the static pressure at B. Show your calculation.

4. Use Bernoulli's equation to find the static pressure at C. Is it higher or lower than at A?

3 YOUR TURN

Your own words

Design a new level pipe section with an inlet area of 10 cm^2, inlet speed of 2 m/s, and inlet pressure of 100,000 Pa. Choose a different outlet area, then use both equations to calculate the outlet speed and pressure; state whether pressure rises or falls. Assume steady water flow, a density of 1,000 kg/m^3, and negligible friction.

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PART 1 · READ AND MARK

Underline the two equations, circle the conditions needed to use them, and number the three pipe sections in flow order. Accept any marking that follows this job and points at the right text.

PART 2 · ANSWERS

#	QUESTION	ANSWER
1	Use the continuity equation to find the water's speed at B. Show your calculation.	**$(8\ cm^2)(2\ m/s) = (4\ cm^2)v_B$, so $v_B = 4\ m/s$.**
2	Use the continuity equation to find the water's speed at C. Show your calculation.	**$(8\ cm^2)(2\ m/s) = (16\ cm^2)v_C$, so $v_C = 1\ m/s$.**
3	Use Bernoulli's equation to find the static pressure at B. Show your calculation.	**$P_B = 120{,}000 + (1/2)(1{,}000)(2^2 - 4^2) = 114{,}000$ Pa.**
4	Use Bernoulli's equation to find the static pressure at C. Is it higher or lower than at A?	**$P_C = 120{,}000 + (1/2)(1{,}000)(2^2 - 1^2) = 121{,}500$ Pa, which is higher than at A.**

PART 3 · YOUR TURN (SAMPLE ANSWER)

I chose an outlet area of 5 cm^2. Continuity gives $(10)(2) = (5)v_{out}$, so the outlet speed is 4 m/s. Bernoulli's equation gives $P_{out} = 100{,}000 + (1/2)(1{,}000)(2^2 - 4^2) = 94{,}000$ Pa. The pressure falls.

Accept any positive outlet area other than 10 cm^2 if the calculated speed and pressure follow both equations. An outlet area below 10 cm^2 must produce a higher speed and lower pressure; an area above it must produce the reverse.