LESSON MAKER PRO

When Lines Meet

Use the structure of equations to predict a system's solutions.

GRADE 8 · 8.AT.B.5.c

8.AT.B.5.c "Use structure to determine when a system has no solution, one solution, or infinitely many solutions and justify the conclusion."

NAME _____ DATE _____

COMPARE THE EQUATIONS

Rewrite each equation as y = mx + b. Different slopes mean one solution, the same slope with different starting values means no solution, and matching slopes and starting values mean infinitely many solutions. When there is one solution, solve for x and then find y.

WORKED EXAMPLE

How many solutions does this system have? Justify your answer. y = 17x + 19 51x - 3y = -57

1. Subtract 51x from both sides of the second equation: -3y = -51x - 57.
2. Divide by -3: y = 17x + 19.
3. Both equations describe the same line.

Answer: Infinitely many solutions, because the equations describe the same line.

PRACTICE 6 problems

1 How many solutions? Justify your answer. y = 4x + 6 y = 4x - 1

ANSWER _____

2 How many solutions? Justify your answer. y = 6x + 4 12x - 2y = -8

ANSWER _____

3 How many solutions? If there is one, give its ordered pair. Justify your answer. y = 5x + 1 y = x + 9

ANSWER _____

4 How many solutions? If there is one, give its ordered pair. Justify your answer. y = -2x + 8 y = x + 2

ANSWER _____

5 How many solutions? Justify your answer. 4x + y = 8 y = -4x + 6

ANSWER _____

6 How many solutions? Justify your answer. 12x + 2y = 10 y = 5 - 6x

ANSWER _____

APPLY IT Show your work

1 Two arcades charge for game rounds. Arcade A charges $4 to enter and $6 per round. Arcade B charges $10 to enter and $4 per round. After how many rounds will the total costs be the same? What will each cost? Write a system and justify your answer.

WORK SPACE

ANSWER _____

2 Two movie-rental plans charge a signup fee plus a cost per movie. Plan A charges $8 to sign up and $4 per movie. Plan B charges $10 to sign up and $4 per movie. Can the plans ever cost the same for the same number of movies? Write a system and justify your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Consider the system $y = 4x + 8$ and $12x - ky = -24$, where k is a number. Find the value of k that makes infinitely many solutions. For every other value of k, find the solution and justify why it is the only one.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	How many solutions? Justify your answer. y = 4x + 6 y = 4x - 1	**No solution. The slopes match, but the starting values differ.**
2	How many solutions? Justify your answer. y = 6x + 4 12x - 2y = -8	**Infinitely many solutions. The second equation rewrites as y = 6x + 4.**
3	How many solutions? If there is one, give its ordered pair. Justify your answer. y = 5x + 1 y = x + 9	**One solution, (2, 11). The slopes differ, and both equations give y = 11 when x = 2.**
4	How many solutions? If there is one, give its ordered pair. Justify your answer. y = -2x + 8 y = x + 2	**One solution, (2, 4). The slopes differ, and both equations give y = 4 when x = 2.**
5	How many solutions? Justify your answer. 4x + y = 8 y = -4x + 6	**No solution. The first equation rewrites as y = -4x + 8, so the slopes match but the starting values differ.**
6	How many solutions? Justify your answer. 12x + 2y = 10 y = 5 - 6x	**Infinitely many solutions. The first equation rewrites as y = 5 - 6x.**

PART 2 · APPLY IT

1. **After 3 rounds, each arcade costs $22.** Let x be rounds and y be total cost. The equations are y = 4 + 6x and y = 10 + 4x. Their slopes differ, and solving 4 + 6x = 10 + 4x gives x = 3 and y = 22.
2. **No. The plans never cost the same for the same number of movies.** Let x be movies and y be total cost. The equations are y = 8 + 4x and y = 10 + 4x. Their slopes match, but their starting values differ.

CHALLENGE

When k = 3, the second equation becomes 12x - 3y = -24, which rewrites as y = 4x + 8, so there are infinitely many solutions. Substitution gives (12 - 4k)(x + 2) = 0. For every k other than 3, this forces x = -2 and y = 0, so the only solution is (-2, 0).

Accept equivalent explanations based on slopes, starting values, or algebraic rewriting. In the challenge, a valid justification must account for every value of k other than 3.