LESSON MAKER PRO

Conic Clues

Rewrite equations to reveal their shapes and key features.

HIGH SCHOOL · A.SSE.B.3.d

A.SSE.B.3.d "Choose and produce an equivalent form of a second degree equation, $Ax^2 + Cy^2 + Dx + Ey + F = 0$, to reveal and explain properties of the conic section represented by the equation."

NAME _____ DATE _____

COMPLETE THE SQUARE

Group the x terms and y terms, factor out each squared-term coefficient, and complete the square. Divide to make the right side 1. Two positive squared terms describe an ellipse, which is a circle if the denominators match; one positive and one negative squared term describe a hyperbola.

WORKED EXAMPLE

Rewrite $16x^2 + 25y^2 - 96x + 100y - 156 = 0$ in standard form. Name the conic and its center.

1. $16(x^2 - 6x) + 25(y^2 + 4y) = 156$
2. $16[(x - 3)^2 - 9] + 25[(y + 2)^2 - 4] = 156$
3. $16(x - 3)^2 + 25(y + 2)^2 = 400$
4. $(x - 3)^2/25 + (y + 2)^2/16 = 1$

Answer: Ellipse with center (3, -2).

PRACTICE 6 problems

1 Rewrite $x^2 + y^2 - 2x - 4y - 4 = 0$ in standard form. Give the center and radius.

ANSWER _____

2 Rewrite $x^2 + y^2 + 6x - 10y + 18 = 0$ in standard form. Give the center and radius.

ANSWER _____

3 Rewrite $4x^2 + 9y^2 - 16x + 18y - 11 = 0$ in standard form. Give the center and the direction of the longer axis.

ANSWER _____

4 Rewrite $9x^2 + 4y^2 + 36x - 16y + 16 = 0$ in standard form. Give the center and the direction of the longer axis.

ANSWER _____

5 Rewrite $x^2 - 4y^2 - 6x - 16y - 11 = 0$ in standard form. Give the center and vertices.

ANSWER _____

6 Rewrite $4y^2 - x^2 + 8x + 16y - 4 = 0$ in standard form. Give the center and vertices.

ANSWER _____

APPLY IT Show your work

1 On a festival map, x and y are measured in meters. The boundary of a circular Wi-Fi zone follows $x^2 + y^2 - 14x + 2y + 1 = 0$. Rewrite the equation to find the router's location at the center and the zone's radius.

WORK SPACE

ANSWER _____

2 An oval LED dance floor has a boundary modeled by $4x^2 + 9y^2 - 24x + 18y - 99 = 0$, with x and y measured in feet. Rewrite the equation to find the floor's center, full width in the x direction, and full width in the y direction.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Rewrite $9x^2 - 4y^2 + 18x + 16y - 43 = 0$ in standard form. Give its vertices, then explain why its branches open left and right even though the y term has the larger denominator.

WORK SPACE AND ANSWER

ANSWER KEY

Conic Clues

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite $x^2 + y^2 - 2x - 4y - 4 = 0$ in standard form. Give the center and radius.	**$(x - 1)^2 + (y - 2)^2 = 9$; center (1, 2), radius 3.**
2	Rewrite $x^2 + y^2 + 6x - 10y + 18 = 0$ in standard form. Give the center and radius.	**$(x + 3)^2 + (y - 5)^2 = 16$; center (-3, 5), radius 4.**
3	Rewrite $4x^2 + 9y^2 - 16x + 18y - 11 = 0$ in standard form. Give the center and the direction of the longer axis.	**$(x - 2)^2/9 + (y + 1)^2/4 = 1$; center (2, -1), longer axis horizontal.**
4	Rewrite $9x^2 + 4y^2 + 36x - 16y + 16 = 0$ in standard form. Give the center and the direction of the longer axis.	**$(x + 2)^2/4 + (y - 2)^2/9 = 1$; center (-2, 2), longer axis vertical.**
5	Rewrite $x^2 - 4y^2 - 6x - 16y - 11 = 0$ in standard form. Give the center and vertices.	**$(x - 3)^2/4 - (y + 2)^2 = 1$; center (3, -2), vertices (1, -2) and (5, -2).**
6	Rewrite $4y^2 - x^2 + 8x + 16y - 4 = 0$ in standard form. Give the center and vertices.	**$(y + 2)^2 - (x - 4)^2/4 = 1$; center (4, -2), vertices (4, -3) and (4, -1).**

PART 2 · APPLY IT

1. **$(x - 7)^2 + (y + 1)^2 = 49$; router at (7, -1), radius 7 meters.** Completing both squares gives a sum of squared distances equal to 49. The center is (7, -1), and the radius is the square root of 49.
2. **$(x - 3)^2/36 + (y + 1)^2/16 = 1$; center (3, -1), x-direction width 12 feet, y-direction width 8 feet.** Complete the squares to get $4(x - 3)^2 + 9(y + 1)^2 = 144$, then divide by 144. Double the square roots of 36 and 16 to get the full widths.

CHALLENGE

$(x + 1)^2/4 - (y - 2)^2/9 = 1$; vertices (-3, 2) and (1, 2). The positive squared term is the x term, so the branches open along the x direction.

Accept equivalent standard forms with terms in either order. For the challenge, the explanation must use the sign of the squared term, not the size of its denominator, to identify the opening direction.