LESSON MAKER PRO

Shape Shift Equations

Rewrite circle and ellipse equations in standard and general form.

HIGH SCHOOL · A.SSE.B.3.e

A.SSE.B.3.e "Translate between the standard and general form, $Ax^2 + Cy^2 + Dx + Ey + F = 0$, of a second degree equation."

NAME _____ DATE _____

THE BIG IDEA

To change standard form to general form, clear fractions, expand the squares, and move every term to one side. To go back, group the x terms and y terms, then complete each square. For an ellipse, divide so the right side is 1. For a circle, leave the sum of the squares equal to the radius squared.

WORKED EXAMPLE

Rewrite $x^2 + y^2 + 14x - 22y + 145 = 0$ in standard form.

1. Group terms: $(x^2 + 14x) + (y^2 - 22y) = -145$.
2. Complete each square: $(x^2 + 14x + 49) + (y^2 - 22y + 121) = -145 + 49 + 121$.
3. Simplify the right side: $(x + 7)^2 + (y - 11)^2 = 25$.

Answer: $(x + 7)^2 + (y - 11)^2 = 25$

PRACTICE 6 problems

1 Rewrite $(x - 3)^2 + (y + 2)^2 = 4$ in general form.

ANSWER _____

2 Rewrite $x^2 + y^2 + 4x - 6y + 4 = 0$ in standard form.

ANSWER _____

3 Rewrite $(x - 1)^2/9 + (y + 1)^2/4 = 1$ in general form with integer coefficients.

ANSWER _____

4 Rewrite $x^2 + 4y^2 - 2x + 16y + 13 = 0$ in standard form.

ANSWER _____

5 Rewrite $3x^2 + 3y^2 - 12x + 6y - 12 = 0$ in standard form.

ANSWER _____

6 Rewrite $(x + 2)^2/16 + (y - 1)^2/4 = 1$ in general form with integer coefficients.

ANSWER _____

APPLY IT Show your work

1 At a gaming event, the edge of a Wi-Fi signal on the floor follows $x^2 + y^2 - 6x + 2y - 6 = 0$, where x and y are measured in meters. Rewrite the equation in standard form. What are the center and radius of the coverage area?

WORK SPACE

ANSWER _____

2 An oval light pattern for a school livestream has boundary $(x - 1)^2/9 + (y + 2)^2/4 = 1$, where x and y are measured in meters. Rewrite its equation in general form with integer coefficients and no common factor.

WORK SPACE

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CHALLENGE One step up

A classmate rewrites $8x^2 + 2y^2 - 32x + 12y + 18 = 0$ as $(x - 2)^2/4 + (y + 3)^2/4 = 1$. Find the correct standard form and explain the error.

WORK SPACE AND ANSWER

ANSWER KEY # Shape Shift Equations

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite $(x - 3)^2 + (y + 2)^2 = 4$ in general form.	**$x^2 + y^2 - 6x + 4y + 9 = 0$**
2	Rewrite $x^2 + y^2 + 4x - 6y + 4 = 0$ in standard form.	**$(x + 2)^2 + (y - 3)^2 = 9$**
3	Rewrite $(x - 1)^2/9 + (y + 1)^2/4 = 1$ in general form with integer coefficients.	**$4x^2 + 9y^2 - 8x + 18y - 23 = 0$**
4	Rewrite $x^2 + 4y^2 - 2x + 16y + 13 = 0$ in standard form.	**$(x - 1)^2/4 + (y + 2)^2 = 1$**
5	Rewrite $3x^2 + 3y^2 - 12x + 6y - 12 = 0$ in standard form.	**$(x - 2)^2 + (y + 1)^2 = 9$**
6	Rewrite $(x + 2)^2/16 + (y - 1)^2/4 = 1$ in general form with integer coefficients.	**$x^2 + 4y^2 + 4x - 8y - 8 = 0$**

PART 2 · APPLY IT

1. **$(x - 3)^2 + (y + 1)^2 = 16$; center (3, -1) meters; radius 4 meters** Complete the x and y squares by adding 9 and 1 to both sides, giving a right side of 16. Read the center and radius from the standard form.
2. **$4x^2 + 9y^2 - 8x + 36y + 4 = 0$** Multiply both sides by 36, expand the squares, and move 36 to the left. Combine the constants: 4 + 36 - 36 = 4.

CHALLENGE

$8(x - 2)^2 + 2(y + 3)^2 = 32$, so the correct form is $(x - 2)^2/4 + (y + 3)^2/16 = 1$. The classmate used 4 for both denominators, but the squared terms have different coefficients. Dividing the equation by 32 gives denominators 32/8 = 4 and 32/2 = 16.

Accept equivalent general forms obtained by multiplying every coefficient by the same nonzero number. For ellipse standard forms, accept the two terms in either order.