LESSON MAKER PRO

Sums Without End

Find when an infinite geometric series has a finite sum.

HIGH SCHOOL · A.SSE.B.5

A.SSE.B.5 "Determine the sum, if it exists, of an infinite geometric series."

NAME _____ DATE _____

THE SUM RULE

In a geometric series, each term is the previous term multiplied by the same number, called the common ratio r. If $|r| < 1$, the infinite series has sum $S = a/(1 - r)$, where a is the first term. If $|r| \geq 1$ and a is not zero, there is no finite sum.

WORKED EXAMPLE

Find the sum of $15 + 5 + 5/3 + \ldots$.

1. The first term is $a = 15$.
2. Divide the second term by the first: $r = 5/15 = 1/3$.
3. Since $|r| < 1$, use $S = a/(1 - r)$.
4. $S = 15/(1 - 1/3) = 15/(2/3) = 45/2$.

Answer: 45/2

PRACTICE 6 problems

1 Find the sum of $8 + 4 + 2 + \ldots$.

ANSWER _____

2 Find the sum of $24 - 6 + 3/2 - \ldots$.

ANSWER _____

3 Does $3 + 9 + 27 + \ldots$ have a finite sum? Explain using the common ratio.

ANSWER _____

4 Find the sum of $11 + 11/4 + 11/16 + \ldots$.

ANSWER _____

5 Does $6 - 9 + 27/2 - \ldots$ have a finite sum?

ANSWER _____

6 Find the sum of the terms $32(1/4)^n$ for $n = 2, 3, 4, \ldots$.

ANSWER _____

APPLY IT Show your work

1 You spend 48 minutes editing a video on Monday. Each following day, you spend 1/4 as much time editing as you did the day before. If this pattern continues forever, how much time do you spend editing in all?

WORK SPACE

ANSWER _____

2 A pair of wireless earbuds gains 36% of a full charge during the first minute of charging. During each later minute, it gains half as much charge as it did in the previous minute. According to this model, what percent of a full charge does it gain in all?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An infinite geometric series starts with 18 and has a sum of 12. Find its common ratio and its next two terms. Explain why the sum exists.

WORK SPACE AND ANSWER

ANSWER KEY # Sums Without End

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the sum of 8 + 4 + 2 + ...	**16**
2	Find the sum of 24 - 6 + 3/2 -	**96/5**
3	Does 3 + 9 + 27 + ... have a finite sum? Explain using the common ratio.	**No finite sum; r = 3, so |r| ≥ 1.**
4	Find the sum of 11 + 11/4 + 11/16 + ...	**44/3**
5	Does 6 - 9 + 27/2 - ... have a finite sum?	**No finite sum.**
6	Find the sum of the terms $32(1/4)^n$ for n = 2, 3, 4, ...	**8/3**

PART 2 · APPLY IT

1. **64 minutes** The first term is 48 and the common ratio is 1/4. The sum is 48/(1 - 1/4) = 64 minutes.
2. **72% of a full charge** The first term is 36% and the common ratio is 1/2. The sum is 36%/(1 - 1/2) = 72%.

CHALLENGE

The common ratio is -1/2, and the next two terms are -9 and 9/2. Since |-1/2| < 1, the series has a finite sum.

Accept equivalent exact fractions or decimals for finite sums. For the challenge, students may find the ratio by solving 12 = 18/(1 - r).