LESSON MAKER PRO

Tale of Tails

Predict where polynomial graphs go beyond the grid.

HIGH SCHOOL · F.IF.C.7.g

F.IF.C.7.g "Determine the end behavior of the graph of a polynomial function using the degree and leading coefficient."

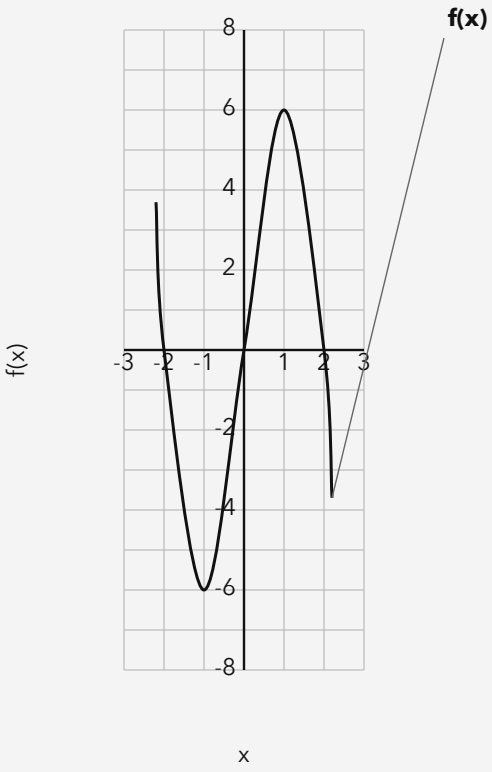
NAME _____ DATE _____

FOLLOW THE LEADING TERM

The highest power in a polynomial gives its degree, and the number multiplying that term is its leading coefficient. An even degree makes both ends go the same way; an odd degree makes them go opposite ways. A positive leading coefficient sends the right end up, while a negative one sends it down.

WORKED EXAMPLE

Look at the graph of $f(x) = -2x^3 + 8x$. Use the leading term to predict both ends of the graph, then check the visible tails.

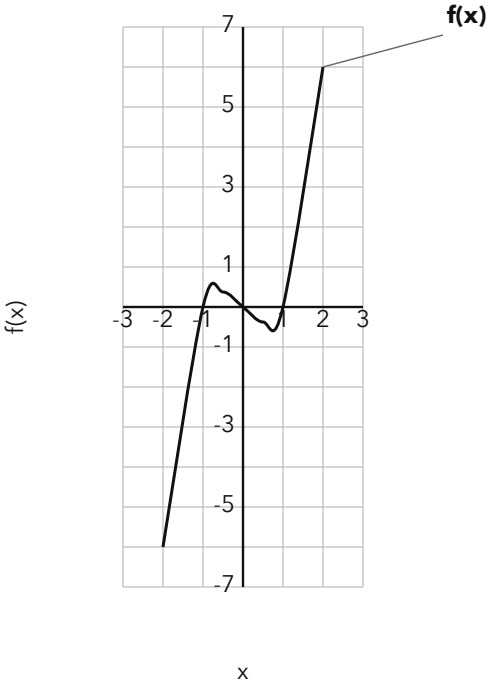


1. The leading term is $-2x^3$.
2. The degree is odd, so the ends go opposite ways.
3. The leading coefficient is negative, so the right end goes down. The left end goes up.

Answer: As $x \to -\infty$, $f(x) \to \infty$. As $x \to \infty$, $f(x) \to -\infty$. The visible tails agree.

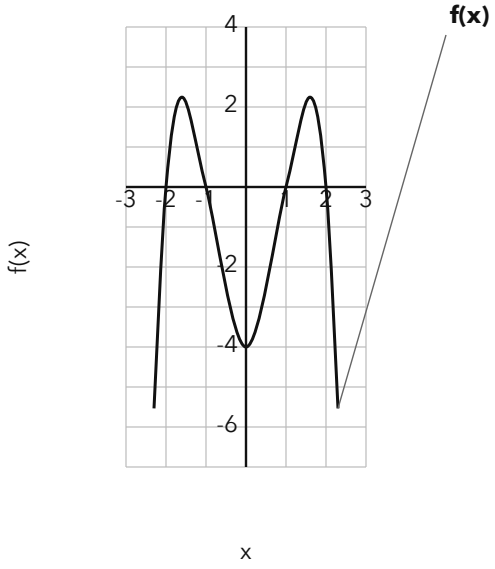
PRACTICE 6 problems

1 For the graphed function $f(x) = x^3 - x$, use its degree and leading coefficient to predict both ends. Do the visible tails agree?



ANSWER _____

2 For the graphed function $f(x) = -x^4 + 5x^2 - 4$, use its degree and leading coefficient to predict both ends. Do the visible tails agree?



ANSWER _____

3 Determine the end behavior of $f(x) = 6x^4 - x^2 + 7$. State its degree and leading coefficient.

ANSWER _____

4 Determine the end behavior of $f(x) = -5x^5 + 3x^4 - 7$. State its degree and leading coefficient.

ANSWER _____

5 Determine the end behavior of $f(x) = 4x - 8x^6 + 1$. State its degree and leading coefficient.

ANSWER _____

6 Determine the end behavior of $f(x) = 5 - 3x^4 + 2x^9$. State its degree and leading coefficient.

ANSWER _____

APPLY IT Show your work

1 In a video game, the vertical position of a moving background is modeled by $y = -3x^4 + 6x^2 + 10$, where x is horizontal position. If the digital background extended without limit in both directions, where would the model send each end?

WORK SPACE

ANSWER _____

2 For a concert's digital backdrop, the vertical position of a light trail is modeled by $y = 2x^5 - 7x^2 + 1$, where x is horizontal position. If the design extended without limit to the left and right, where would the model send each end?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = (a - 4)x^7 - (a + 1)x^6 + 2x^2$, find every value of a for which both ends of the graph fall. Explain why the degree matters.

WORK SPACE AND ANSWER

ANSWER KEY

Tale of Tails

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For the graphed function $f(x) = x^3 - x$, use its degree and leading coefficient to predict both ends. Do the visible tails agree?	**Degree 3, positive leading coefficient. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to \infty$. Yes, the visible tails agree.**
2	For the graphed function $f(x) = -x^4 + 5x^2 - 4$, use its degree and leading coefficient to predict both ends. Do the visible tails agree?	**Degree 4, negative leading coefficient. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to -\infty$. Yes, the visible tails agree.**
3	Determine the end behavior of $f(x) = 6x^4 - x^2 + 7$. State its degree and leading coefficient.	**Degree 4, leading coefficient 6. As $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to \infty$.**
4	Determine the end behavior of $f(x) = -5x^5 + 3x^4 - 7$. State its degree and leading coefficient.	**Degree 5, leading coefficient -5. As $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to -\infty$.**
5	Determine the end behavior of $f(x) = 4x - 8x^6 + 1$. State its degree and leading coefficient.	**Degree 6, leading coefficient -8. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to -\infty$.**
6	Determine the end behavior of $f(x) = 5 - 3x^4 + 2x^9$. State its degree and leading coefficient.	**Degree 9, leading coefficient 2. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to \infty$.**

PART 2 · APPLY IT

1. **The model sends both ends down: as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to -\infty$.** The leading term is $-3x^4$. Its degree is even and its leading coefficient is negative, so both ends fall.
2. **The model sends the left end down and the right end up: as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.** The leading term is $2x^5$. Its degree is odd and its leading coefficient is positive, so the ends go in opposite directions, with the right end rising.

CHALLENGE

$a = 4$. Then the x^7 term disappears, leaving $-5x^6$ as the leading term. Its degree is even and its leading coefficient is negative, so both ends fall. For any other value of a, the degree is 7 and the ends go in opposite directions.

Accept verbal descriptions such as "left down, right up" in place of limit notation. In the challenge, the explanation must account for the change in degree when $a = 4$.